

# A wake-up call: Contagion through alertness\*

Toni Ahnert<sup>†</sup>, Christoph Bertsch<sup>‡</sup>

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## Abstract

We provide a novel contagion mechanism based on an *alertness effect*. Observing an adverse event is a *wake-up call* that induces investors to acquire costly information about the potential exposure to that event. Even if the exposure to the adverse event turns out to be absent, information acquisition in itself can trigger fragility. Our mechanism is applicable to sovereign debt crises, bank runs, currency attacks, political regime change, and other coordination problems. The contagion-through-alertness mechanism offers an explanation for the 1997 Asian currency crisis where financial fragility spread to countries with seemingly unrelated fundamentals and limited inter-connectedness.

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<sup>†</sup>London School of Economics and Political Science, Financial Markets Group and Department of Economics, Houghton Street, London WC2A 2AE, United Kingdom. Part of this research was conducted when the author was visiting the Department of Economics at New York University and the Federal Reserve Board of Governors. Email: [t.ahnert@lse.ac.uk](mailto:t.ahnert@lse.ac.uk).

<sup>‡</sup>Department of Economics, University College London, Gower Street, London WC1E 6BT, United Kingdom. Email: [c.bertsch@ucl.ac.uk](mailto:c.bertsch@ucl.ac.uk)

# 1 Introduction

This paper provides a novel contagion mechanism in coordination problems. Observing an adverse event in another country is a *wake-up call* for an investor who may be exposed to that event. Consequently, an investor acquires costly information about her exposure. We call information acquisition after an adverse event elsewhere an *alertness effect* and demonstrate that it can act as a catalyst for contagion. Relative to not learning about the exposure, financial fragility can be *higher* when the exposure is small. This contagion-through-alertness effect even prevails after investors learn that their investments are completely unrelated to the adverse event. Presenting an application to speculative currency attacks, we offer an explanation for the 1997 Asian currency crisis, where financial fragility spread to countries with seemingly unrelated fundamentals and limited interconnectedness. Our contagion-through-alertness mechanism occurs generally in coordination problems and is also applicable to bank runs, political regime change, and sovereign debt crises.

There are several examples of adverse events that potentially spill over to other regions. Bank creditors of one bank observe a run on another bank. Likewise, currency speculators in one market observe a successful attack on another currency. In the Arab spring, political activists in one country observe a political regime change in another country. Common to these examples is that agents are not directly affected by the adverse event but might be affected indirectly. The adverse event carries information about an agent's payoffs if fundamentals are correlated across regions, resulting in information contagion. In the bank run example correlation between fundamentals can arise because of system-wide liquidity and solvency problems after fire

sales.<sup>1</sup> In the currency attack example correlation between fundamentals can arise from a limited ability or willingness to defend a currency union, some indication of which is observed in the European sovereign debt crisis.<sup>2</sup>

The adverse event is a *wake-up call* for directly unaffected agents. While it is ex-ante unknown whether fundamentals are correlated, there is some chance that the correlation is positive and agents are detrimentally affected. Consequently, agents wish to determine the extent of their exposure to the adverse event by acquiring costly information. Information acquisition after receiving the wake-up call from the other region's adverse event is an *alertness effect*. For example, one bank's debt holder is alerted to the potential spillover from a run on another bank and acquires information about exposure of her bank to the insolvent bank or about whether both banks invested in the same asset classes.

Information acquisition in itself can result in financial fragility. Even if investors learn that their investment is completely independent of the adverse event, there can be more financial fragility than in the case where investors are uninformed. Thus, fragility in one region can lead to fragility in a second region even if fundamentals are independent, a *contagion-through-alertness* effect.

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<sup>1</sup>The pressure on banks to deleverage in a financial crisis can initiate a downward spiral in the prices of assets held by other banks. This creates correlated liquidity and solvency problems in the banking system.

<sup>2</sup>A growing common exposure of European countries emerged through the European Financial Stability Facility (EFSF), the European Stabilisation Mechanism (ESM), and the European System of Central Banks (ESCB). However, rescue funds are limited and the commitment to full sharing of liabilities is questionable such that a sovereign default of euro zone country remains possible. Such a sovereign default would create significant losses for other euro member states. Losses arise via burden sharing to creditor countries after the first default and via insufficient rescue funds for further bail outs to other countries close to default.

The fragility in the second region is a direct consequence of the information acquisition of investors and arises in the context of coordination problems. The additional information about the correlation of fundamentals has two effects. First, it leads to a change in the posterior mean of the regional fundamental (*mean effect*). Having observed an adverse event, an investor's posterior mean improves upon learning that fundamentals are uncorrelated. Second, it affects the variance of the posterior distribution of the fundamental in the second region (*variance effect*). The coordination problem between investors in the second region can be worsened, implying a larger degree of coordination failure. When the variance effect dominates the mean effect, the good news of no correlation with the adverse event implies *more* fragility in the second region. In sum, we show that contagion can occur even if investors learn that they are not exposed to the adverse event.

An investor has an ex-ante incentive to acquire information whenever the cost of information is sufficiently low. Intuitively, the information on the correlation of fundamentals helps an investor improve her forecast about the regional fundamental as well as the behaviour of other investors. We demonstrate that an investor earns a higher gross expected payoff after adjusting her strategy because of being informed. We apply our mechanism to currency attacks where fundamentals represent the government's ability to defend the currency. We show that an informed speculator obtains a higher expected payoff than an uninformed speculator by acting more (less) aggressively after receiving information that lowers (improves) their forecast for fundamentals. By doing so, an informed speculator increases her expected benefits from participating in a successful currency attack when fundamentals are weak.

Likewise, an informed speculator reduces her expected costs of participating in a unsuccessful currency attack when fundamentals are strong.

Our contagion mechanism is general and lends itself to several applications. It provides a new and compelling explanation of the contagious spread of the Asian currency crisis in 1997. One country's currency was attacked after the other's, despite seemingly unrelated fundamentals and limited exposure between these countries. The same holds for the popular example of Brazil during the Russian crisis of 1998. Although Brazil had very limited exposure, it was still one of the most affected countries (see Pavlova and Rigobon [23]). Furthermore, a strong co-movement of international asset prices does not require correlated fundamentals or interconnectedness. Finally, the contagion-through-alertness mechanism applies generally to coordination problems in which the payoff from acting depends on both the underlying state of the world and the proportion of other agents acting. Alternative applications are foreign direct investment across emerging markets (see Dasgupta [10]) and the aforementioned Arab spring, in which political activists observe a revolution in a neighbouring country and decide whether or not to attempt a revolution themselves (see Edmond [11]).

Our paper is related to Morris and Shin [17, 19] who develop an incomplete information game of speculative currency attacks in the tradition of the global games literature pioneered by Carlsson and van Damme [6]. We differ in two main aspects. First, we consider a two-country model with potentially correlated fundamentals and address the issue of contagion across countries. Currency speculators move sequentially such that speculators in the second region decide whether to attack their regional currency after observing the

outcome in the first region (wake-up call). Second, speculators in the second region can endogenously acquire information about the correlation of fundamentals after observing the outcome in the first region (alertness effect). Speculators can be asymmetrically informed as in Corsetti et al. [7] but the information asymmetry is endogenous in our model.<sup>3</sup> While these papers do not address contagion, our focus is on demonstrating that contagion through alertness can take place even if fundamentals are uncorrelated.

We share with Goldstein and Pauzner [13] the focus on coordination failure in financial crisis and the analysis of contagion. The authors obtain contagion because of risk aversion of speculators invested in both regions. After a crisis in the first region speculators become more averse to strategic risk and have a larger incentive to withdraw their investment.<sup>4</sup> By contrast, information is endogenous in our setting. We consider the unintended consequences of ex-ante optimal information acquisition on financial fragility via the contagion-through-alertness mechanism. Since speculators are risk-neutral in our model, we provide a complementary contagion mechanism.

This paper is organised as follows. The model is described in section 2 and solved in section 3. Section 4 solves for the optimal decision whether to acquire information. A more general discussion of the related literature is offered in section 5. Finally, section 6 concludes. All proofs and most derivations are relegated to the appendices.

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<sup>3</sup>The authors also consider different sizes of currency speculators and its effect on the incidence and severity of a currency crisis. Our speculators are of equal size as our contagion-through-alertness mechanism does not require signalling or herding.

<sup>4</sup>The authors also consider the ambivalent welfare implications of diversification, highlighting unintended consequences of diversification.

## 2 Model

Consider an **incomplete information game** with two dates  $t \in \{1, 2\}$  and two regions indexed by the date of taking action. Each region is inhabited by a continuum of risk-neutral agents  $i \in [0, 1]$  interpreted as currency speculators.<sup>5</sup> At each date speculators in a given region **act simultaneously**. The **action space** is **binary**. Each agent has two possible actions  $a_{it} \equiv a_i \in \{0, 1\}$ , where action  $a_i = 1$  refers to a currency attack and action  $a_i = 0$  to no currency attack.

Both regions are characterised by a region-specific **fundamental**  $\theta_t$  that measures the government's strength to defend its currency, such as its foreign reserves. Fundamentals are bivariate normally distributed with mean  $\mu_t \equiv \mu \in (0, 1)$  and precision  $\alpha_t \equiv \alpha > 0$ .<sup>6</sup> The correlation between regional fundamentals is initially uncertain. It is independently drawn from a uniform distribution:

$$\rho \sim U[0, \rho_H] \tag{1}$$

where imperfect correlation between fundamentals  $\rho_H \in (0, 1)$  ensures that agents who learn the correlation are still imperfectly informed. Before choosing her action, each agent receives a private signal about her region's fundamental:

$$x_{it} \equiv \theta_t + \epsilon_{it} \tag{2}$$

Idiosyncratic noise  $\epsilon_{it}$  is identically and independently normally distributed across agents and regions with zero mean and precision  $\gamma$  and is also inde-

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<sup>5</sup>Speculators may act nationally or internationally and might thus be the same across regions. All we require is that agents in different regions act sequentially.

<sup>6</sup>In the complete-information version of the game  $\mu \in (0, 1)$  ensures multiplicity of equilibria. If  $\mu \leq 0$ , then  $a_i^* = 1$  is strictly dominant; if  $\mu \geq 1$ , then  $a_i^* = 0$  is strictly dominant.

pendent of fundamentals as well the correlation of fundamentals. All distributions are common knowledge.

The success of a currency attack depends on both the fundamental  $\theta_t$  and the proportion of attacking agents denoted by  $A_t \equiv \int_0^1 a_{it} di$ . The **payoff** of an attacking speculator is:

$$u(a_{it} = 1, A_t, \theta_t) \equiv A_t - \theta_t \tag{3}$$

as in Morris and Shin [19]. An agent's payoff from not attacking is constant and normalised to zero:  $u(a_{it} = 0) \equiv 0$ . A currency attack is successful if  $A_t \geq \theta_t$ .

Speculators in region 2 observe whether there was a successful currency attack in region 1. We assume that they observe  $\theta_1$  directly for simplicity. The game at date  $t = 2$  has **two stages**. Before taking her action in a coordination game at a second stage, each agent in region 2 may purchase a signal about the correlation of the fundamentals at cost  $c > 0$ . The signal is common to all purchasers and publicly available. A figurative example of such a signal is a newspaper, which takes money to buy and time to absorb. In terms of wholesale investors or currency speculators, the access to Bloomberg and Datastream terminals and hiring analysts to interpret the publicly available information comes to mind. Information acquisition is costly in each case. The signal about the correlation of fundamentals is perfectly revealing for simplicity again. The proportion of agents who acquire information at stage 1 is publicly observed.<sup>7</sup>

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<sup>7</sup>This can be relaxed and our results do not depend on this.



The following **timeline** summarises the model:

**Date  $t = 1$**

- The fundamental  $\theta_1$  is drawn; each agent receives a private signal  $x_{i1}$ .
- Agents in region 1 decide **simultaneously** whether or not to act.
- The fundamental  $\theta_1$  is publicly observed by agents in both regions and payoffs are realised.

**Date  $t = 2$**

- **Stage 1:** Agents in region 2 **simultaneously** decide whether or not to purchase a publicly available signal about the correlation of fundamentals  $\rho$  at cost  $c > 0$ . The proportion of agents who are informed  $n \in [0, 1]$  is publicly observable.
- **Stage 2:**
  - The fundamental  $\theta_2$  is drawn; each agent receives a private signal  $x_{i2}$ .
  - Agents in region 2 decide **simultaneously** whether or not to act.
  - The fundamental  $\theta_2$  is publicly observed and payoffs are realised.

### 3 Equilibrium

The focus of this paper is on the equilibrium in region 2. In particular, we describe how events in region 1 influence this equilibrium, contrasting the situation of known and unknown correlation between fundamentals. It is therefore useful to revise briefly the equilibrium in region 1, which is a standard coordination game as in e.g. Morris and Shin [19] such that a detailed derivation is relegated to Appendix A.1.

### 3.1 Region 1

A given speculator  $i$  in region 1 uses her private signal to update her beliefs about the fundamental of region 1:

$$\theta_1|x_{i1} \sim \mathcal{N}\left(\Theta_{i1}, \frac{1}{\alpha + \gamma}\right) \quad (4)$$

$$\Theta_{i1} \equiv \frac{\alpha\mu + \gamma x_{i1}}{\alpha + \gamma} \quad (5)$$

where  $\Theta_{i1}$  is speculator  $i$ 's posterior mean of region 1's fundamental. The equilibrium threshold posterior mean  $\Theta_1^*$  is implicitly defined by the indifference between attacking and not attacking the currency:

$$\Phi\left(\sqrt{\delta_0}[\Theta_1^* - \mu]\right) = \Theta_1^* \quad (6)$$

$$\delta_0 \equiv \frac{\alpha^2(\alpha + \gamma)}{\gamma(\alpha + 2\gamma)}$$

There is a unique Bayesian equilibrium in threshold strategies. The threshold is unique and given by the solution to (6) if  $\delta_0 < 2\pi$ . This is a common sufficient condition in global games and requires the private signal to be sufficiently precise relative to the public signal (prior distribution). The unique Bayesian equilibrium in threshold strategies prescribes a speculator to attack the currency if and only if  $\Theta_{i1} \leq \Theta_1^*$ , that is when the government's foreign reserves are sufficiently low. Applying an argument of iterated deletion of strictly dominant strategies as in Morris and Shin [19], there is no profitable unilateral deviation from the threshold strategy with threshold  $\Theta_1^*$ . Given that all speculators follow this threshold strategy, a currency attack is successful if the regional fundamental is below the unique

equilibrium threshold ( $\theta_1 < \theta_1^*$ ) implicitly defined by:

$$\theta_1^* = \Phi \left( \frac{\alpha}{\sqrt{\gamma}} [\Theta_1^* - \mu] + \sqrt{\gamma} [\Theta_1^* - \theta_1^*] \right) \quad (7)$$

There are two possible rankings of the equilibrium thresholds depending on the prior mean of the fundamental. First, fundamentals are called *weak* when  $0 < \mu < \frac{1}{2}$ . Weak fundamentals lead to strong attacks on the currency:  $\mu < \frac{1}{2} < \Theta_1^* < \theta_1^* < 1$ , implying little coordination failure. Currency attacks may occur for fundamentals stronger than the prior mean. Second, fundamentals are called *strong* when  $\frac{1}{2} \leq \mu < 1$ . Strong fundamentals lead to less frequent currency attacks:  $0 < \theta_1^* < \Theta_1^* < \frac{1}{2} \leq \mu$ , implying a large degree of coordination failure. These rankings are for finite private noise ( $\gamma < \infty$ ); refer to Appendix A.1.2 for a detailed analysis. Finally, Appendix A.1.3 describes how the likelihood of currency attacks changes with the precision of public information.

## 3.2 Region 2

The game at date  $t = 2$  is *solved backwards*. We consider the two stages in reverse order. The second stage in region 2 is an extension of the equilibrium in region 1 to asymmetrically informed speculators. It is discussed in this section and the main results are summarised in Proposition 1. Next, we establish the our novel contagion effect in section 3.3 for an exogenous amount of information. Finally, we solve for the first stage and endogenise information acquisition in section 4.

We introduce some notation to distinguish informed and uninformed speculators. Using the subscripts  $I$  and  $U$ , respectively, the private signal of informed (uninformed) speculator  $i$  in region 2 is given by  $x_{i2I}$  ( $x_{i2U}$ ). Further-

more, let  $a_{i2I}$  ( $a_{i2U}$ ) denote the action of informed (uninformed) speculator  $i$  in region 2, let  $A_{2I}$  ( $A_{2U}$ ) denote the fraction of attacking speculators that are informed (uninformed), and let  $\Theta_{i2I}$  ( $\Theta_{i2U}$ ) denote the posterior mean of informed (uninformed) speculator  $i$  in region 2.

**Updating** A mass  $n \in [0, 1]$  of speculators is informed. An **informed** speculator updates her prior distribution as follows, taking the correlation of fundamentals into account ( $\rho \geq 0$ ):

$$\Theta_{2I}(\rho) \sim \mathcal{N}\left(\rho\theta_1 + (1 - \rho)\mu, \frac{1 - \rho^2}{\alpha}\right) \quad (8)$$

The posterior distribution of fundamentals in region 2 converges to the prior distribution in region 1 if fundamentals are uncorrelated:  $\theta_2 \sim \mathcal{N}(\mu, \frac{1}{\alpha})$ .

An **uninformed** speculator observes  $\theta_1$  and uses the prior distribution of the correlation  $\rho$  to update her belief about the fundamental in region 2. The distribution  $\theta_2|\theta_1$  is derived in Appendix section A.2 and given by:

$$\Theta_{2U} \sim \mathcal{N}\left(\mu - \rho_H \frac{\mu - \theta_1}{2}, \frac{1 - \frac{\rho_H^2}{3}}{\alpha}\right) \quad (9)$$

With a uniform prior about the correlation of fundamentals, the posterior distribution is similar to a sum of normally distributed random variables and therefore also normally distributed. The posterior mean differs from the prior mean  $\mu$  and shifts towards  $\theta_1$  the further, the larger the correlation. The posterior mean is smaller than the prior mean if a currency attack is observed in region 1 and fundamentals are strong ( $\theta_1 < \theta_1^* < \frac{1}{2} < \mu$ ). The posterior precision unambiguously increases.

We focus on values of  $\theta_1$  that preserve the coordination failure problem. For that purpose we required  $\mu > 0$  in region 1. In the informed case, we require  $\rho\theta_1 + (1 - \rho)\mu > 0$  for any  $\rho \in [0, \rho_H]$ . In the uninformed case, we require  $\mu - \rho_H \frac{\mu - \theta_1}{2} > 0$ . A sufficient condition for these inequalities is:

$$\theta_1 > \left(1 - \frac{1}{\rho_H}\right) \mu \quad (10)$$

which is the less binding, the smaller  $\rho_H$ .

We can now compare the posterior distributions between informed and uninformed speculators. Consider the specific case of a low fundamental realisation in region 1 ( $\theta_1 < \mu$ ); the case for high fundamentals is symmetric. An informed speculator has a higher mean than the uninformed speculator if and only if  $\rho < \frac{\rho_H}{2}$ . Similarly, an informed speculator has a lower variance than the uninformed speculator if and only if  $\rho > \frac{\rho_H}{\sqrt{3}}$ . In sum, the informed speculator has both a higher mean and a higher variance than the uninformed speculator if  $\rho < \frac{\rho_H}{2}$ , a lower mean and a higher variance if  $\frac{\rho_H}{2} < \rho < \frac{\rho_H}{\sqrt{3}}$ , and a lower mean and a lower variance if  $\frac{\rho_H}{\sqrt{3}} < \rho$ . Our contagion mechanism occurs for zero correlation ( $\rho = 0$ ) and we will see how the higher relative mean and variance contribute to our effect.

### 3.2.1 Special case $n = 0$ : classical information contagion

This special case of completely uninformed speculators captures the classical information contagion channel, which is distinct from our novel contagion mechanism to be established in section 3.3. Bad news in region 1 translate into bad news in the region 2 as fundamentals may be positively correlated. The strength of this effect is measured by the maximum amount of correla-

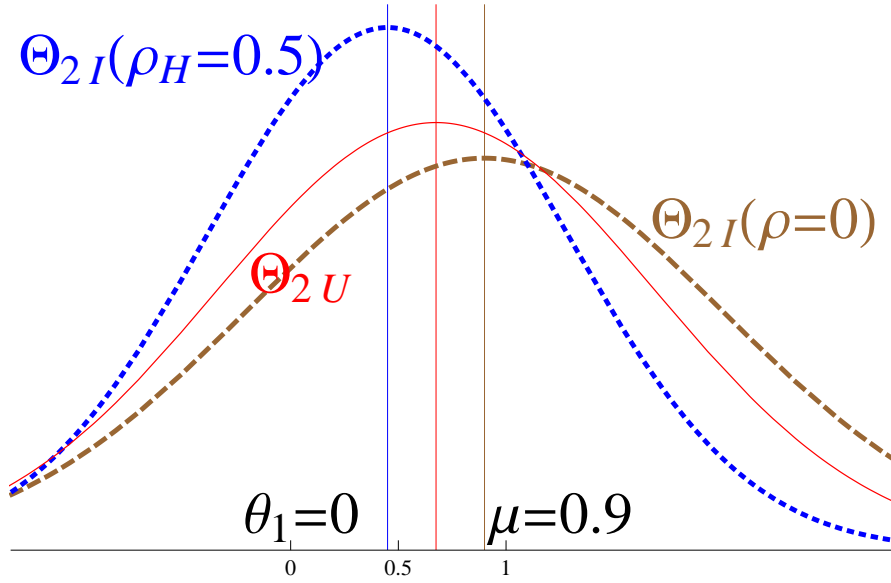


Figure 1: Higher (lower) posterior mean and variance for informed speculators if  $\rho = 0$  (if  $\rho = 0 = \rho_H$ ).

tion ( $\rho_H$ ).<sup>8</sup>

We show that there exists again a unique equilibrium in threshold strategies as in our analysis of region 1. See Appendix A.3.1 for a detailed analysis, where the equilibrium and sufficient conditions for its uniqueness are derived. A successful currency attack occurs in region 2 if the regional fundamental is below its unique threshold ( $\theta_2 < \theta_{2U}^* \in (0, 1)$ ). The posterior mean  $\Theta_{2U}^*$  and the threshold for the regional fundamental  $\theta_{2U}^*$  are implicitly defined by:

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<sup>8</sup>An example of this information contagion channel is Acharya and Yorulmazer [1] who show that funding costs of one bank increase after bad news about another bank if the banks' loan portfolio returns have a common factor.

$$\Theta_{2U}^* = \Phi \left( \sqrt{\delta_1} [\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}] \right) \quad (11)$$

$$\theta_{2U}^* = \Phi \left( \frac{\alpha}{\sqrt{\gamma}(1 - \rho_H^2/3)} [\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}] + \sqrt{\gamma}(\Theta_{2U}^* - \theta_{2U}^*) \right) \quad (12)$$

$$\delta_1 \equiv \frac{\alpha^2(\alpha + \gamma(1 - \rho_H^2/3))}{\gamma(1 - \rho_H^2/3)^2(\alpha + 2\gamma(1 - \rho_H^2/3))} \quad (13)$$

Observe that the classical information contagion mechanism is present. The equilibrium threshold of the fundamental decreases in the fundamental observed in region 1 ( $\partial\theta_{2U}^*/\partial\theta_1 < 0$ ). A lower observed fundamental in region 1 implies that the fundamental in region 2 is likely to be low as well, implying little defense by the government against a currency attack. Consequently, it is optimal for speculators to attack the currency in region 2 for a larger region of posterior means, thus raising the equilibrium threshold of the fundamental in region 2 below which a currency attack is successful.

### 3.2.2 The general case $0 < n < 1$

In the general case we allow for asymmetrically informed speculators. A fraction  $n$  of speculators learns the inter-regional correlation  $\rho$  (informed speculators), while a fraction  $1 - n$  of speculators does not learn the correlation (uninformed). As before speculators use threshold strategies, where uninformed speculators attack if their posterior mean is below a threshold  $\Theta_{2U}^*$  and informed speculators attack if their posterior mean is below a threshold  $\Theta_{2I}^*(\rho)$ . Note that there is a sequence of thresholds for informed speculators  $\{\Theta_{2I}^*(\rho)\}$ , one for each realisation of the correlation of fundamentals. These thresholds are determined by the usual indifference conditions between attacking and not attacking the currency and are derived in Appendix A.3.2.

**Proposition 1** *If private signals are sufficiently precise ( $\gamma > \underline{\gamma}$ ), then there exists a unique equilibrium in threshold strategies in this subgame. Each uninformed speculator withdraws if and only if her posterior mean is smaller than the threshold  $\Theta_{2U}^*$ . Each informed speculator, who observes the correlation of fundamentals  $\rho$ , withdraws if and only if her posterior mean is smaller than the threshold  $\Theta_{2I}^*(\rho)$ . The threshold of uninformed speculators  $\Theta_{2U}^*$  and the sequence of equilibrium thresholds of informed speculators  $\Theta_{2I}^*(\rho)$  are implicitly given by:*

$$\Theta_{2I}^*(\rho) = n\Phi\left(\sqrt{\delta_2}[\Theta_{2I}^*(\rho) - \rho\theta_1 - (1-\rho)\mu]\right) \quad (14)$$

$$+ (1-n)\Phi\left(\sqrt{\delta_3}\left[\frac{\alpha}{\gamma(1-\rho_H^2/3)}(\Theta_{2U}^* - \mu + \rho_H\frac{\mu - \theta_1}{2}) + \Theta_{2U}^* - \Theta_{2I}^*(\rho)\right]\right)$$

$$\Theta_{2U}^* = n \int_0^{\rho_H} \frac{\Phi\left(\sqrt{\delta_4}\left[\frac{\alpha}{\gamma(1-\tilde{\rho}^2)}(\Theta_{2I}^*(\tilde{\rho}) - \tilde{\rho}\theta_1 - (1-\tilde{\rho})\mu) + \Theta_{2I}^*(\tilde{\rho}) - \Theta_{2U}^*\right]\right)}{\rho_H} d\tilde{\rho}$$

$$+ (1-n)\Phi\left(\sqrt{\delta_1}[\Theta_{2U}^* - \mu + \rho_H\frac{\mu - \theta_1}{2}]\right) \quad (15)$$

where:

$$\delta_2 \equiv \frac{\alpha^2(\alpha + \gamma(1 - \rho^2))}{(1 - \rho^2)^2\gamma(\alpha + 2\gamma(1 - \rho^2))} \quad (16)$$

$$\delta_3 \equiv \frac{\gamma(\alpha + \gamma(1 - \rho^2))}{\alpha + 2\gamma(1 - \rho^2)} \quad (17)$$

$$\delta_4 \equiv \frac{\gamma(\alpha + \gamma(1 - \rho_H^2/3))}{\alpha + 2\gamma(1 - \rho_H^2/3)} \quad (18)$$

*Given that all speculators follow this threshold strategy, the threshold of the regional fundamental  $\theta_2^*$  below which there is a successful currency attack*



$(\theta_2 < \theta_2^*)$  is:

$$\theta_2^* = n\Phi\left(\frac{\alpha}{\sqrt{\gamma}(1-\rho^2)}(\Theta_{2I}^*(\rho) - [\rho\theta_1 + (1-\rho)\mu]) + \sqrt{\gamma}(\Theta_{2I}^*(\rho) - \theta_2^*)\right) \quad (19)$$

$$+(1-n)\Phi\left(\frac{\alpha}{\sqrt{\gamma}(1-\rho_H^2/3)}(\Theta_{2U}^* - [\mu - \rho_H\frac{\mu - \theta_1}{2}]) + \sqrt{\gamma}(\Theta_{2U}^* - \theta_2^*)\right)$$

**Proof** See Appendix A.4.

As in region 1, the usual global games condition of sufficiently high precision of the private signal to the public signal (prior distribution) ensures uniqueness. Because of the additional strategic interaction between informed and informed speculators, the relative precision needs to be higher than in region 1 or in the polar case of  $n = 0$  in region 2.

### 3.3 Contagion through alertness

Suppose there was a successful currency attack in the first region, such that the ability of the government in region 1 to defend the currency must have been low. If fundamentals are positively correlated across regions, the government's ability to defend is likely to be also low in region 2, therefore making a successful currency attack likely to take place in region 2 as well. However, and perhaps surprisingly, the incidence of successful currency attacks can be *higher* if speculators learn that fundamentals are little (or not at all correlated) than if speculators do not learn about the correlation.

In particular we demonstrate in this section that the *ex-ante* incidence of speculative attacks when all speculators are informed and learn that fundamentals are uncorrelated ( $n = 1, \rho = 0$ ) can be higher than the *ex-ante* incidence of attacks when all speculators are uninformed ( $n = 0$ ). Here *ex-*

*ante* refers to the beginning of stage 2, that is before  $\theta_2$  is realised. We call this effect contagion through alertness (to the successful currency attack in region 1). Our result holds generally in coordination failure problems. For example, observing a bank run in one bank can lead to a bank run in another, even after the depositors of the second bank learned that their banks has little exposure to the first one.

Learning good news might have detrimental effects. Learning good news is bad when it increases the variance of the posterior distribution relative to the case of not learning any news. The variance matters despite risk neutrality as knowing what others do is payoff-relevant information in coordination problems. This effect via the variance of the posterior distribution may lead to contagion across regions via heightened coordination failure.

Suppose that a fraction of speculators  $n \in [0, 1]$  is informed and receives interim information (at stage 1 of date 2) about the correlation of the fundamentals. For the time being  $n$  is exogenously determined and we consider the incentives for information acquisition after observing a successful currency attack in region 1 in section 4 as an extension. While our result holds more generally, the special polar cases in which all speculators are uninformed ( $n = 0$ ) and all speculators are informed ( $n = 1$ ) help to build intuition. The contagion effect can be present for strong fundamentals and therefore a large degree of coordination failure. To ensure an equivalent ranking as for region 1, where we had  $\theta_1 < \theta_1^* < \mu$  for strong fundamentals ( $\frac{1}{2} < \mu < 1$ ), the posterior mean of region 2 needs to lie in  $(\frac{1}{2}, 1)$  as well, which implies the

following bounds on  $\theta_1$ :

$$\theta_1 \in \left( \mu + \frac{1 - 2\mu}{\rho_H}, \mu + \frac{2 - 2\mu}{\rho_H} \right) \quad (20)$$

Taking the equilibrium as characterised in the previous section and considering the special polar cases, we have:

**Special case  $n = 0$**

$$\Theta_{2U}^* = \Phi \left( \sqrt{\delta_1} [\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}] \right) \quad (21)$$

$$\theta_{2U}^* = \Phi \left( \frac{\alpha}{\sqrt{\gamma}(1 - \rho_H^2/3)} [\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}] + \sqrt{\gamma} [\Theta_{2U}^* - \theta_{2U}^*] \right) \quad (22)$$

**Special case  $n = 1, \rho = 0$**  (Note that  $\delta_2(\rho = 0) = \delta_0$ .)

$$\Theta_{2I}^*(0) = \Phi \left( \sqrt{\delta_0} [\Theta_{2I}^*(0) - \mu] \right) \quad (23)$$

$$\theta_{2I}^*(0) = \Phi \left( \frac{\alpha}{\sqrt{\gamma}} [\Theta_{2I}^*(0) - \mu] + \sqrt{\gamma} [\Theta_{2I}^*(0) - \theta_{2I}^*(0)] \right) \quad (24)$$

We are interested in uncovering when the ex-ante incidence of currency attacks is higher upon learning that fundamentals are uncorrelated, that is when  $\Theta_{2I}^*(\rho = 0) > \Theta_{2U}^*$ . Inspecting the equilibrium conditions reveals that there are two effects at work: a *mean effect* and a *variance effect*.

The *mean effect* occurs when the informed speculator has a lower posterior mean relative to the uninformed speculator, which happens if  $\rho > \frac{\rho_H}{2}$ . Then, the mean effect implies a higher threshold for informed speculators:  $\Theta_{2I}^* > \Theta_{2U}^*$ . Instead, for independent fundamentals ( $\rho = 0$ ), the mean effect works against our desired result. Choosing the average mean,  $\rho = \frac{\rho_H}{2}$ , that uninformed speculators also face, the mean effect is zero. The mean effect

applies to both weak and strong fundamentals and holds as long as  $\theta_1 < \mu$ .

The *variance effect* refers to a larger variance of the posterior distribution for informed speculators relative to uninformed speculators. This happens when  $\rho < \frac{\rho_H}{\sqrt{3}}$  for strong fundamentals. Then, we obtain the desired result of  $\Theta_{2I}^* > \Theta_{2U}^*$ . Note that the variance effect is independent of the realisation of  $\theta_1$  and symmetric since it also holds when  $\rho > \frac{\rho_H}{\sqrt{3}}$  for weak fundamentals. The variance effect can also be shut down, which occurs for the “average variance” of  $\rho = \frac{\rho_H}{\sqrt{3}}$ .

In sum, the desired result of  $\Theta_{2I}^* > \Theta_{2U}^*$  obtains for independent and strong fundamentals if the variance effect is sufficiently strong relative to the mean effect. We have the following proposition:

**Proposition 2** *The effect of **contagion through alertness** exists for strong fundamentals if:*

$$\frac{\sqrt{\delta_1} - \sqrt{\delta_0}}{\sqrt{\delta_1}} > \frac{\rho_H[\mu - \theta_1]}{2\mu} \leq \frac{2\mu - 1}{2} \quad (25)$$

**Proof** Inspecting the equilibrium conditions yields the following sufficient condition for the desired result:

$$(\sqrt{\delta_1} - \sqrt{\delta_0})[\mu - \Theta_{2I}^*] > \frac{\rho_H}{2}\sqrt{\delta_1}[\mu - \theta_1] \quad (26)$$

The result follows by using  $\Theta_{2I}^* \geq 0$ .

The sufficient conditions contains three terms. The variance effect is represented by the first term on the left hand side. There is a higher variance when learning that fundamentals are uncorrelated:  $\delta_1 > \delta_0$ . The mean effect

is represented by the two terms on the right-hand side. The mean effect is proportional to the average correlation  $\frac{\rho_H}{2}$  and the stronger, the larger the upper bound on correlation. The mean effect is also proportional to  $\frac{\mu-\theta_1}{\mu}$ , which reflects the updating towards  $\theta_1$ . The mean effect is strongest if the fundamentals in region 1 are weak, that is if  $\theta_1$  is small.

The contagion through alertness effect can only be present for strong fundamentals. The upper bound of the right-hand side of equation (26) is positive for strong fundamentals only. The left-hand side is always positive. We obtain the strong fundamentals as a necessary condition of the effect of contagion through alertness as the left-hand side needs to exceed the right-hand side.

**Intuition** Contagion through alertness can be present even after good news. This happens if the additional coordination failure implied by the higher variance of the posterior distribution weighs more than the change in the mean of the posterior distribution after good news. At the core of the contagion effect through alertness is a higher posterior variance translates into **more strategic uncertainty**. Strategic uncertainty refers to the behaviour of other speculators as perceived by a given speculator and states that there is more coordination failure among speculators.

Take the case of strong fundamentals first. Then, the equilibrium posterior threshold is low. With a high probability speculator  $i$ 's posterior is below the prior mean. She then expects that a given speculator  $j$ 's posterior mean is above hers. This is illustrated in figure 2.

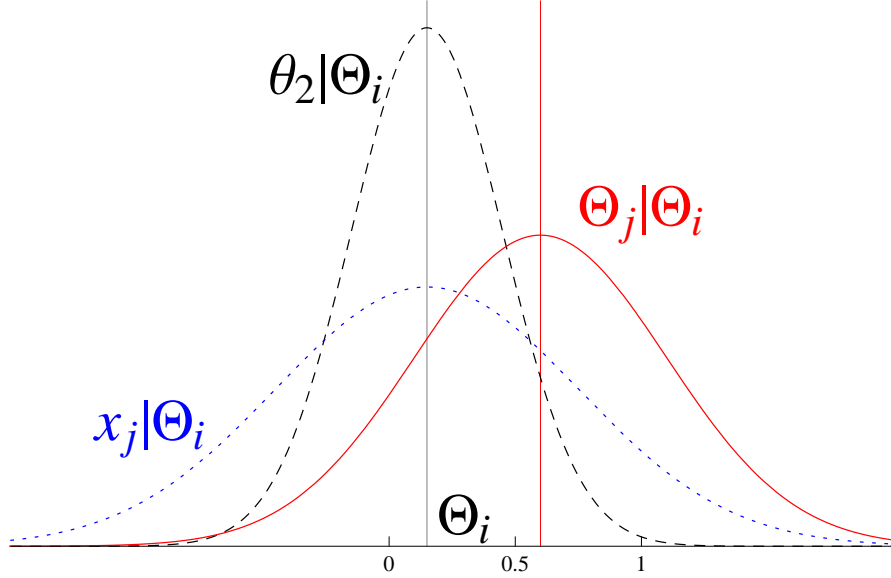


Figure 2: Posterior distribution

We contrast the posterior distributions of informed and uninformed speculators to illustrate the effect of additional variance of the posterior distribution. Speculator  $i$  expects a larger fraction of speculators receiving a signal that corresponds to a lower posterior. Figure 3 shows this effect as an increase in the area under the curve left of  $\Theta'$  for the more dispersed posterior distribution. Therefore, a larger share of informed speculators than uninformed speculators attack the currency despite expecting stronger defense of the currency by the government.

Likewise, the equilibrium posterior mean is above the prior mean for weak fundamentals and a given speculator  $i$  is likely to have a posterior above  $\mu$ . Hence, she expects that a given speculator  $j$ 's posterior mean is lower than hers. In contrast to strong fundamentals, more strategic uncertainty causes

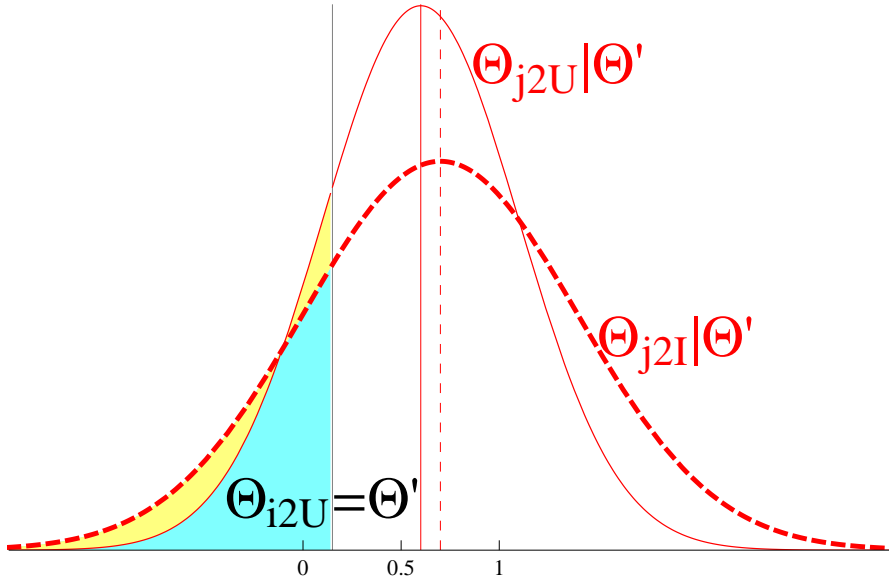


Figure 3: Posterior distribution - strong fundamentals

speculator  $i$  to expect a lower fraction of speculators receiving a signal that corresponds to a lower posterior mean. To see this consider figure 4. The variance effect implies a lower level of equilibrium currency attacks for weak fundamentals, the opposite result to strong fundamentals.

As a result, more strategic uncertainty only causes a higher equilibrium incidence of attacks by informed speculators if fundamentals are strong. Then, learning that fundamentals are uncorrelated reduces the posterior variance and the heightened coordination failure implies contagion of speculative attacks from region 1.

## 4 Information acquisition

We derived the contagion through alertness effect in the previous section. While the amount of information was exogenous – a fraction  $n \in [0, 1]$  was

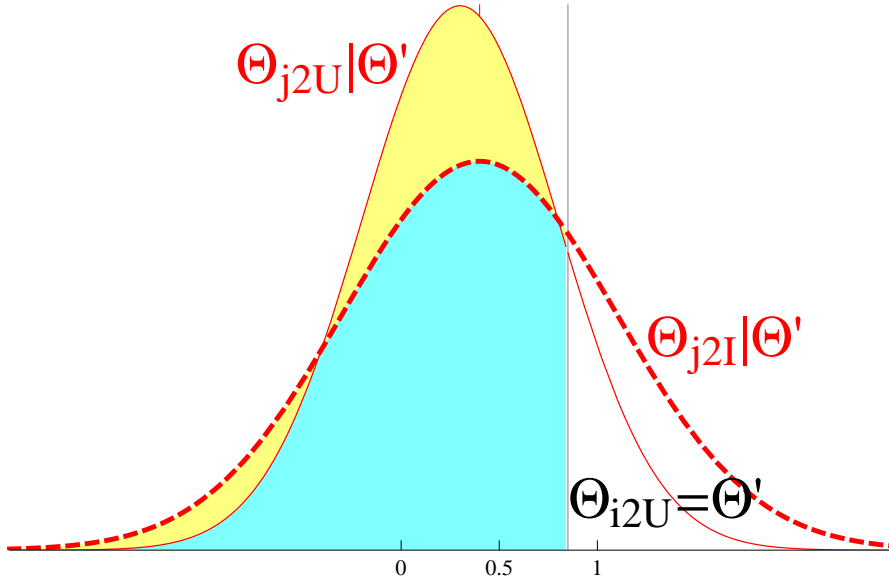


Figure 4: Posterior distribution - weak fundamentals

informed, we allow for endogenous information acquisition in this section and thereby generalise our result. We show that there exists an equilibrium in which each speculator acquires information if the cost of doing so is sufficiently small. The contagion-through-alertness effect is present in this equilibrium: there can be more runs after speculators learn that fundamentals are uncorrelated than without having learned anything.

After observing region 1's fundamental  $\theta_1$ , speculators in region 2 decide whether to acquire costly information on the inter-regional correlation  $\rho$ . Recall that the purchased information is a perfect signal about  $\rho$  for simplicity and that the additional signal is publicly available to all speculators. We maintain our focus on case in which speculators in region 2 observe a crisis in region 1, that is  $\theta_1 < \theta_1^* < \mu$  for strong fundamentals.



**The speculator's problem** To determine the equilibrium of the game, we consider the problem of an individual speculator. Each speculator  $i$  takes the population proportion of speculators  $n$  who purchase information as given and compares the expected payoffs from purchasing the publicly available signal (becoming informed  $s = I$ ) and not purchasing the signal (remaining informed  $s = U$ ). The expected utility from becoming informed  $EU_I$  is:

$$\begin{aligned} EU_I &\equiv \mathbb{E}[u(s = I, \alpha, \gamma, \mu, \rho_H, \theta_1, n)] \\ &= \int_0^{\rho_H} \left( \int_{-\infty}^{+\infty} \Gamma_I(\theta_2, \rho, n) f(\theta_2 | \theta_1, \rho) d\theta_2 \right) \frac{d\rho}{\rho_H} - c \end{aligned} \quad (27)$$

$$\Gamma_I(\theta_2, \rho, n) \equiv \int_{x_{i2} \leq x_{2I}^*(\rho, n)} \left( \begin{array}{l} (1-n) \Pr\{x_{j2} \leq x_{2U}^* | \theta_2\} + \\ n \Pr\{x_{j2} \leq x_{2I}^*(\rho, n) | \theta_2\} - \theta_2 \end{array} \right) g(x | \theta_2) dx \quad (28)$$

The expected utility from remaining uninformed  $EU_U$  is:

$$\begin{aligned} EU_U &\equiv \mathbb{E}[u(s = U, \alpha, \gamma, \mu, \rho_H, \theta_1, n)] \\ &= \int_{-\infty}^{+\infty} \Gamma_U(\theta_2, n) f(\theta_2 | \theta_1) d\theta_2 \end{aligned} \quad (29)$$

$$\Gamma_U(\theta_2, n) \equiv \int_{x_{i2} \leq x_{2U}^*(n)} \left( \begin{array}{l} (1-n) \Pr\{x_{j2} \leq x_{2U}^* | \theta_2\} + \\ n \int_0^{\rho_H} \Pr\{x_{j2} \leq x_{2I}^*(\rho) | \theta_2\} \frac{d\rho}{\rho_H} - \theta_2 \end{array} \right) g(x | \theta_2) dx \quad (30)$$

where the posterior distributions of the fundamental in region 2 (when informed and uninformed) as well as the posterior distribution of signals is given as follows:

$$f(\theta_2 | \theta_1, \rho) = \sqrt{\frac{\alpha}{2\pi(1-\rho^2)}} \exp\left\{-\frac{\alpha}{2(1-\rho^2)} (\theta_2 - (\rho\theta_1 + (1-\rho)\mu))^2\right\} \quad (31)$$

$$f(\theta_2 | \theta_1) = \sqrt{\frac{\alpha}{2\pi(1-\rho_H^2/3)}} \exp\left\{-\frac{\alpha}{2(1-\rho_H^2/3)} \left(\theta_2 - \left(\mu - \rho_H \frac{\mu - \theta_1}{2}\right)\right)^2\right\} \quad (32)$$

$$g(x | \theta_2) = \sqrt{\frac{\gamma}{2\pi}} \exp\left\{-\frac{\gamma}{2}(x - \theta_2)^2\right\} \quad (33)$$

**Intuition** Before the fundamental  $\theta_2$  is realised, speculators know the conditional distribution of  $\theta_2$ . For informed speculators the pdf is given by  $f(\theta_2|\theta_1, \rho)$  and for uninformed speculators by  $f(\theta_2|\theta_1)$ . For each realisation of  $\theta_2$ , speculators can compute how many (un-) informed speculators decide to attack and how likely it is that they themselves receive a private signal below their critical threshold which induces them to attack. The uninformed speculator integrates over the posterior distribution of  $\theta_2$  only, while the informed speculator has a different posterior distribution for each observed correlation. Hence, she integrates over the posterior distribution (inner integral) and the possible outcomes of the correlation (outer integral). Taking together, this results in the expected payoffs in equations (27) and (29).

A given speculator finds it optimal to purchase the publicly available signal if the differential payoff  $\Delta[\alpha, \gamma, \mu, \rho_H, \theta_1, n] \equiv EU_I - EU_U$  is positive.

**Definition** A pure strategy Perfect Bayesian Nash Equilibrium in region 2 is an information acquisition choice  $s_i^* \in \{I, U\}$  for each speculator  $i \in [0, 1]$  in stage 1, a decision rule  $a_{i2s}^*(\theta_1, x_{i2}, n)$  in stage 2 and a fraction of informed speculators  $n^*$  such that:

1. All speculators optimally choose  $s_i$  in stage 1.
2. The proportion  $n^*$  is consistent with the optimal choices implied by (1):  

$$n^* = \int_0^1 \mathbf{1} \{s_i^* = I\} di.$$
3. Each uninformed speculator  $i$  optimally chooses in stage 2:

$$a_{i2U}(\theta_1, x_{i2}, n) = \begin{cases} 1 & \text{if } \Theta_{i2U}(x_{i2}) < \Theta_{2U}^*(n^*) \\ 0 & \text{if } \Theta_{i2U}(x_{i2}) > \Theta_{2U}^*(n^*) \\ \in \{0, 1\} & \text{otherwise} \end{cases} \quad (34)$$

and each informed speculator  $i$  optimally chooses in stage 2:

$$a_{i2I}(\theta_1, x_{i2}, n, \rho) = \begin{cases} 1 & \text{if } \Theta_{i2I}(\rho, x_{i2}) < \Theta_{2I}^*(\rho, n^*) \\ 0 & \text{if } \Theta_{i2I}(\rho, x_{i2}) > \Theta_{2I}^*(\rho, n^*) \\ \in \{0, 1\} & \text{otherwise} \end{cases} \quad (35)$$

where the threshold  $\Theta_{2U}^*(n^*)$  and the sequence of thresholds  $\Theta_{2I}^*(\rho, n^*)$  are implicitly defined by equations (14) and (15), respectively.

4.  $\theta_2^*(n^*)$  solves equation (19).

**Benefits from and costs of attacking** To gain a better understanding of the advantage for a speculator to be informed, consider the benefits and costs from attacking for the special case when  $n = 0$ .

The **uninformed** speculator's benefit from using a strategy with attack threshold  $x_{2U}^*$  is given by:

$$\int_{-\infty}^{\theta_{2U}^*} \left( \int_{x_{i2} \leq x_{2U}^*} (A_{2U}^*(\theta_2) - \theta_2) g(x|\theta_2) dx \right) f(\theta_2|\theta_1) d\theta_2 > 0 \quad (36)$$

and the costs are given by:

$$\int_{\theta_{2U}^*}^{+\infty} \left( \int_{x_{i2} \leq x_{2U}^*} (A_{2U}^*(\theta_2) - \theta_2) g(x|\theta_2) dx \right) f(\theta_2|\theta_1) d\theta_2 < 0 \quad (37)$$

where  $A_{2U}^*(\theta_2) \equiv \Pr\{x_{j2} \leq x_{2U}^*|\theta_2\}$ . Note that costs and benefits only differ in the outer integral. Attacking the currency yields a positive (negative) payoff if the fundamental realisation is below (above) the critical threshold  $\theta_{2U}^*$ .

Similarly, the **informed** speculators' benefits from using the strategy with attack threshold  $x_{2I}^*(\rho)$  are given by:

$$\int_0^{\rho_H} \int_{-\infty}^{\theta_{2U}^*} \left( \int_{x_{i2} \leq x_{2I}^*(\rho)} (A_{2U}^*(\theta_2) - \theta_2) g(x|\theta_2) dx \right) f(\theta_2|\theta_1, \rho) d\theta_2 \frac{d\rho}{\rho_H} > 0 \quad (38)$$

and the costs are given by:

$$\int_0^{\rho_H} \int_{\theta_{2U}^*}^{+\infty} \left( \int_{x_{i2} \leq x_{2I}^*(\rho)} (A_{2U}^*(\theta_2) - \theta_2) g(x|\theta_2) dx \right) f(\theta_2|\theta_1, \rho) d\theta_2 \frac{d\rho}{\rho_H} - c < 0 \quad (39)$$

The benefit from being informed is the ability to better forecast the fundamental  $\theta_2$ . This advantage can be seen intuitively when comparing an uninformed speculator with an informed speculator that observes the lowest possible correlation  $\rho = 0$ , consequently forming an optimistic update about the government's ability to defend the currency. The informed speculator places relatively less weight on  $\theta_2 \in (-\infty, \theta_{2U}^*]$  and relatively more weight on  $\theta_2 \in (\theta_{2U}^*, \infty)$ .<sup>9</sup>

We now ask whether an informed speculator gains from using a more aggressive strategy, that is a strategy with a marginally lower threshold  $x_{2I}^*(\rho_H) = x_{2U}^* - \xi$ , where  $\xi > 0$ . It depends on the cost of information acquisition. Recall that the critical threshold  $x_{2U}^*$  used by uninformed speculators is chosen such that the marginal benefit of increasing the threshold equals the marginal cost, i.e.  $\frac{dEU}{dx_{2U}}|_{x_{2U}=x_{2U}^*} = 0$ . Instead, an informed speculator places less weight on low fundamental realisations and, hence, optimally

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<sup>9</sup>In particular we have that:

$$\frac{df(\theta_2|\theta_1, \rho)}{d\rho} \Big|_{\rho \rightarrow 0} \begin{cases} < 0 & \text{if } \theta_2 > \mu \\ \geq 0 & \text{if } \theta_2 \leq \mu \end{cases} \quad (40)$$

selects a  $x_{2I}^*(\rho = 0) < x_{2U}^*$  as the marginal benefit of decreasing the critical threshold below  $x_{2U}^*$  outweighs the marginal cost. The informed speculator can thereby decrease her expected costs from attacking when the attack is unsuccessful, as given in equation (39), by more than her expected benefits from attacking when the attack is successful, as given in equation (38). Hence, a speculator gains from being informed as long as the cost of information acquisition is sufficiently small. This argument is formalised in Lemma (3), which studies the polar cases  $n = 0$  and  $n = 1$ .

**Lemma 3** *An individual speculator has an incentive to acquire information if she believes that all other speculators:*

(a) *are uninformed ( $n = 0$ ), whenever the cost of acquiring the publicly available signal  $c$  is sufficiently small:*

$$c \leq \bar{c} \equiv \Delta[\alpha, \gamma, \mu, \rho_H, \theta_1, 0] \quad (41)$$

(b) *are informed ( $n = 1$ ), whenever the cost of acquiring the publicly available signal  $c$  is sufficiently small:*

$$c \leq \bar{c} \equiv \Delta[\alpha, \gamma, \mu, \rho_H, \theta_1, 1] \quad (42)$$

**Proof** See Appendix A.5.

**Proposition 4** *If private signals are sufficiently precise and the cost of acquiring the public signal  $c$  satisfies the sufficient condition given in equation (42) with strict equality, then there exists an equilibrium of the game in region 2 in which all speculators acquire the publicly available signal after observing  $\theta_1 < \mu$ . We thus have  $n^* = 1$ .*

**Proof** The result follows from Lemma 3 (b).

Given Proposition (4), we demonstrated that the **contagion-through-alertness effect** described in the previous section is an equilibrium phenomenon in the more general setup with endogenous information acquisition.

## 5 Related literature

The literature on currency crisis is large and we do not attempt to provide a detailed review but focus on the incomplete information game introduced by Morris and Shin [17, 19]. Following the seminal contribution of Carlsson and van Damme [6], a perturbation of the information structure yields a unique equilibrium. This overcomes the multiplicity of equilibria present in many previous models of currency crisis, such as the Krugman-Flood-Garber [16, 12] first-generation currency crisis model, the second-generation currency crisis model by Obstfeld [21], and many third-generation currency crisis models.

Our paper is also related to the literature analysing the role of information precision. Information acquisition can have a detrimental effect in our model. This result connects to papers that stress the possible benefits of coarse information.<sup>10</sup> For instance, the papers of Dang, Gorton and Holmström [8] as well as Pagano and Volpin [22] emphasise the benefits of coarse information in supporting market liquidity.

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<sup>10</sup>See Morris and Shin [20].

A key ingredient of our contagion-through-alertness mechanism is the exacerbation of the coordination problem when the precision of the agents' prior information changes. This element is present in earlier work on bank runs by Rochet and Vives [24], for example. In combination with endogenous information acquisition, however, this element gives rise to our novel contagion mechanism. Specifically, investors in our model choose whether to acquire additional information about the exposure to a crisis elsewhere.

Endogenous information acquisition is considered by Hellwig and Veldkamp [14] who discuss the similarity in the strategic motives between choosing an action and deciding on how much information to acquire in a beauty-contest model. In their words, investors “who want to do what others do, want to know what others know” (p. 223). They show that adding a public information choice may lead to a multiplicity in equilibria. By contrast, uniqueness is always guaranteed under the usual mild condition of sufficiently precise private signals in our global games model.

## **Contagion in financial economics**

While there exists a large literature on financial contagion, typically either interconnectedness or common exposures is required to generate contagion or systemic fragility more generally. First, systemic fragility because of common exposures (correlated fundamentals) are considered in Acharya and Yorulmazer [1], who show that banks can have an ex-ante incentive to correlate their investment decision to avoid information contagion, and Allen, Babus and Carletti [3], who analyze systemic risk resulting from the interaction of common exposures and funding maturity through an information channel. Second, financial contagion can arise from interconnectedness. Allen and

Gale [4] provide a model of financial contagion as an equilibrium outcome through interbank linkages.<sup>11</sup> In Goldstein and Pauzner [13] contagion results from a wealth effect of investors who become more averse to strategic risk after a crisis in one country. There is also a large literature on contagion through a pecuniary “fire-sale” externality related to the ideas of Shleifer and Vishny [25].

The distinct feature of the proposed contagion-through-alertness mechanism is the endogenous information acquisition such that contagion can occur in the absence of interconnectedness and common exposures. Observing an adverse event in another region is a wake-up call to investors that induces them to acquire costly information about their exposure to that event. This alertness effect can result in a higher likelihood of an adverse event in their region. Such fragility can even be present if investors learn that their investments are completely uncorrelated with the adverse event. In sum, it is sufficient that fundamentals are potentially correlated to generate the alertness effect. Once speculators are alert, the incidence of speculative attacks is increased even after speculators learn that the regional fundamentals are uncorrelated.

## **Contagion in international finance**

The international finance literature mainly considers a terms-of-trade channel and a common-discount-factor channel to explain an international co-movement in asset prices during crisis periods. (Co-movement in asset prices is considered as contagious when “excessive”). However, these channels

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<sup>11</sup>Dasgupta [9] shows that financial contagion arises with positive probability in a global-game version of Allen and Gale [4]



cannot account for the observed co-movements in the 1997/1998 emerging market crisis period. Pavlova and Rigobon [23] argue that neither channel explains the co-movements in asset prices of countries with limited trade links. They construct an open-economy dynamic stochastic general equilibrium model and show that portfolio constraints can cause a substantial amplification and help to explain the observed co-movements in asset prices in crisis periods. An alternative amplification mechanism is provided by Kodres and Pritsker [15] who establish the “cross-market portfolio rebalancing channel”, which is based on the common discount factor channel.

Calvo and Mendoza [5] also offer a contagion mechanism that does not rely on correlated macroeconomic fundamentals, where the authors relate contagion to information acquisition. In this sense their paper is closer to our model than the existing mechanisms in the financial economics literature. In their paper a lower degree of information acquisition, as a consequence of globalisation, gives rise to contagion because market participants prefer imitate arbitrary market portfolios instead of gathering information which can lead to a detrimental herding behaviour. By contrast, contagion is a consequence of a higher, not a lower, degree of information acquisition in our model. We place an *alertness effect*, that is information acquisition after observing a crisis elsewhere, at the heart of our mechanism, where fragility can arise because of heightened strategic uncertainty in coordination problems.

## 6 Conclusion

This paper proposes a novel contagion mechanism based on an *alertness effect*. Upon observing a crisis elsewhere – a *wake-up call*, investors wish to

determine the extent to what their investment position is affected by that crisis. This alertness effect per se can lead to a larger incidence of crisis through an increase in strategic uncertainty. The contagion-through-alertness effect takes place when investors learn to have small exposures to that crisis and prevails even if the investment of investors is *completely independent* of the crisis. While we present an application to speculative currency attacks, the contagion-through-alertness mechanism occurs in general coordination problems and is applicable to bank runs, political regime change, and sovereign debt crises.

# A Appendix

## A.1 Region 1

### A.1.1 Equilibrium analysis

The coordination game in region 1 is standard and follows [19]. Notice that speculator  $i$ 's posterior mean of region 1's fundamental, given in equation (5), is a one-to-one mapping from her signal  $x_{i1}$ . It is therefore convenient to use the posterior mean  $\Theta_{i1}$  instead of the original signal in what follows. The distribution of another speculator's signal  $x_{j1}$  conditional on the the posterior mean of speculator  $i$  is:

$$x_{j1}|\Theta_{i1} \sim \mathcal{N}\left(\Theta_{i1}, \left[\frac{\gamma(\alpha + \gamma)}{(\alpha + 2\gamma)}\right]^{-1}\right) \quad (43)$$

Conditional on  $\Theta_{i1}$ , the share of agents receiving a lower signal  $x_{j1} < x_{i1}$  or, equivalently, forming a smaller posterior mean ( $\Theta_{j1} < \Theta_{i1}$ ) is the probability of the following event:  $\Pr\{\Theta_{j1} < \Theta_{i1}|\Theta_{i1}\}$ . The expected share of agents receiving a lower signal than the equilibrium signal is thus:

$$\Pr\{\Theta_{j1} < \Theta_{i1}|\Theta_{i1}\} = \Phi\left(\sqrt{\delta_0}[\Theta_{i1} - \mu]\right) \quad (44)$$

$$\delta_0 \equiv \frac{\alpha^2(\alpha + \gamma)}{\gamma(\alpha + 2\gamma)} \quad (45)$$

$$\Rightarrow \Pr\{\Theta_{j1} < \Theta_1^*|\Theta_1^*\} = \Phi\left(\sqrt{\delta_0}[\Theta_1^* - \mu]\right) = E[A_1|\Theta_1^*] \quad (46)$$

where the last line evaluates the expected share at the equilibrium posterior mean to be determined, thereby providing a formula for the equilibrium posterior mean of the share of attacking speculators.

The equilibrium threshold posterior mean  $\Theta_1^*$  is implicitly defined by an indifference between attacking and not attacking:

$$\begin{aligned} E[u(a_i = 1, A_1, \theta_1)|\Theta_1^*] &= E[A_1 - \theta_1|\Theta_1^*] = 0 = E[u(a_i = 1, A_1, \theta_1)|\Theta_1^*] \\ \Rightarrow \Phi\left(\sqrt{\delta_0}[\Theta_1^* - \mu]\right) &= \Theta_1^* \end{aligned} \quad (48)$$

Agent  $i$  attacks if and only if her posterior mean is below the critical threshold:

$$a_{i1}^* = 1 \Leftrightarrow \Theta_{i1} < \Theta_1^* \quad (49)$$

Using equation (5) the threshold signal is:

$$x_1^* = \frac{(\alpha + \gamma)\Theta_1^* - \alpha\mu}{\gamma} \quad (50)$$

Finally, we obtain the threshold of the regional fundamental  $\theta_1^*$ : there is a currency attack for realised fundamentals below this threshold,  $\theta_1 < \theta_1^*$ . For a given fundamental  $\theta_1$ :

$$x_{i1} \sim \mathcal{N}\left(\theta_1, \frac{1}{\gamma}\right) \quad (51)$$

$$\Theta_{i1} \sim \mathcal{N}\left(\frac{\alpha\mu + \gamma\theta_1}{\alpha + \gamma}, \frac{\gamma}{(\alpha + \gamma)^2}\right) \quad (52)$$

In equilibrium:

$$\theta_1^* = A_1^* = \int_0^1 a_{i1}^* di = \int_0^1 \mathbf{1}\{\Theta_{i1} < \Theta_1^*\} di = \Pr\{\Theta_{i1} < \Theta_1^*\} \quad (53)$$

$$\Rightarrow \theta_1^* = \Phi\left(\frac{\alpha}{\sqrt{\gamma}}[\Theta_1^* - \mu] + \sqrt{\gamma}[\Theta_1^* - \theta_1^*]\right) \quad (54)$$

The left-hand side strictly increases in  $\theta_1$  and the right-hand side strictly decreases in it. Also, the right-hand side exceeds the left hand side at  $\theta_1 = 0$

and vice versa at  $\theta_1 = 1$ . Therefore, there exists a unique threshold of fundamentals  $\theta_1^* \in (0, 1)$ . The probability of a currency crisis is thus  $\Pr\{\theta_1 \leq \theta_1^*\} = \Phi(\sqrt{\alpha}[\theta_1^* - \mu])$ . As idiosyncratic noise vanishes,  $\gamma \rightarrow \infty$ ,  $\Theta_1^* = \theta_1^* = \frac{1}{2}$ .

### A.1.2 Equilibrium characterization

Consider equation (6). It is a continuous function in  $\mu$  and we have that  $\Theta_1^* = \frac{1}{2}$  if  $\mu = \frac{1}{2}$  and  $\Theta_1^* > \frac{1}{2}$  ( $\Theta_1^* < \frac{1}{2}$ ) if  $\mu \rightarrow 0$  ( $\mu \rightarrow 1$ ). Further, notice that the stronger the fundamental, the lower the withdrawal threshold:

$$\frac{d\Theta_1^*}{d\mu} = \frac{-\sqrt{\delta_0}\phi(\cdot)}{1 - \sqrt{\delta_0}\phi(\cdot)} < 0 \quad (55)$$

As a result, we have that  $\Theta_1^* > \mu$  ( $\Theta_1^* < \mu$ ) if  $0 < \mu < \frac{1}{2}$  ( $1 > \mu > \frac{1}{2}$ ). Together with equation (7) we have that for weak fundamentals ( $0 < \mu < \frac{1}{2}$ ) there are strong attacks on the currency:  $\mu < \frac{1}{2} < \Theta_1^* < \theta_1^* < 1$ . For strong fundamentals ( $\frac{1}{2} \leq \mu < 1$ ), however, attacks on the currency are less frequent:  $0 < \theta_1^* < \Theta_1^* < \frac{1}{2} \leq \mu$ .

These inequalities are proven by contradiction. First,  $\Theta_1^* < \theta_1^*$  is proven by contradiction. Suppose that  $\Theta_1^* \geq \theta_1^*$ . Observe that  $\frac{\alpha}{\sqrt{\gamma}}[\Theta_1^* - \mu] > 0$  and  $\sqrt{\gamma}[\Theta_1^* - \theta_1^*] > 0$  in equation (7). Furthermore,  $\sqrt{\delta_0}[\Theta_1^* - \mu] > 0$  in equation (6). However, a necessary condition for  $\Theta_1^* \geq \theta_1^*$  is that:

$$\sqrt{\frac{\alpha^2(\alpha + \gamma)}{\gamma(\alpha + 2\gamma)}}[\Theta_1^* - \mu] \geq \frac{\alpha}{\sqrt{\gamma}}[\Theta_1^* - \mu] + \sqrt{\gamma}[\Theta_1^* - \theta_1^*] \quad (56)$$

which leads to again to a contradiction. Second,  $\Theta_1^* > \theta_1^*$  is proven by contradiction. Suppose that  $\Theta_1^* \leq \theta_1^*$ . Observe that  $\frac{\alpha}{\sqrt{\gamma}}[\Theta_1^* - \mu] < 0$  and  $\sqrt{\gamma}[\Theta_1^* - \theta_1^*] < 0$  in equation (7). Furthermore,  $\sqrt{\delta_0}[\Theta_1^* - \mu] < 0$  in equation

(6). However, a necessary condition for  $\Theta_1^* \leq \theta_1^*$  is that:

$$\sqrt{\frac{\alpha^2(\alpha + \gamma)}{\gamma(\alpha + 2\gamma)}}[\Theta_1^* - \mu] \leq \frac{\alpha}{\sqrt{\gamma}}[\Theta_1^* - \mu] + \sqrt{\gamma}[\Theta_1^* - \theta_1^*] \quad (57)$$

which leads to a contradiction, as  $\sqrt{\frac{\alpha^2(\alpha + \gamma)}{\gamma(\alpha + 2\gamma)}} < \frac{\alpha}{\sqrt{\gamma}}$ .

### A.1.3 Higher precision of the public signal ( $\alpha$ )

In this section we consider the effect of an exogenous increase in the public signal's precision, e.g. because of government communication. Notice that we only consider increases in  $\alpha$  that do not violate the sufficiency condition of equilibrium uniqueness. First, it shows that:

$$\frac{\partial \delta_0}{\partial \alpha} = \frac{\alpha(2\alpha^2 + 7\alpha\gamma + 4\gamma^2)}{\gamma(\alpha + 2\gamma)^2} > 0 \quad (58)$$

Therefore, the equilibrium withdrawal threshold falls as public information becomes more precise if fundamentals are strong:

$$\frac{\partial \Theta_1^*}{\partial \alpha} = \frac{0.5\delta_0^{-0.5}[\Theta_1^* - \mu]\phi(\sqrt{\delta_0}[\Theta_1^* - \mu])\frac{\partial \delta_0}{\partial \alpha}}{1 - \sqrt{\delta_0}\phi(\sqrt{\delta_0}[\Theta_1^* - \mu])} \leq 0 \quad \text{if } \frac{1}{2} \leq \mu < 1 \quad (59)$$

$$> 0 \quad \text{if } 0 < \mu < \frac{1}{2} \quad (60)$$

where  $1 - \sqrt{\delta_0}\phi(\cdot) > 0$  by the sufficiency condition for uniqueness. Consequently, the threshold of the fundamental at which a successful currency attack occurs also falls if fundamentals are strong:

$$\frac{\partial \theta_1^*}{\partial \alpha} = \frac{\phi(z_1)}{1 + \sqrt{\gamma}\phi(z_1)} \left( \frac{\alpha + \gamma}{\sqrt{\gamma}} \frac{\partial \Theta_1^*}{\partial \alpha} + \frac{\Theta_1^* - \mu}{\sqrt{\gamma}} \right) \quad (61)$$

$$z_1 = \frac{\alpha}{\sqrt{\gamma}}[\Theta_1^* - \mu] + \sqrt{\gamma}[\Theta_1^* - \theta_1^*] \quad (62)$$

where:

$$\frac{\partial \theta_1^*}{\partial \alpha} \leq 0 \quad \text{if } \frac{1}{2} \leq \mu < 1 \quad (63)$$

$$> 0 \quad \text{if } 0 < \mu < \frac{1}{2} \quad (64)$$

In sum, more transparency, a higher public precision, unambiguously makes currency attacks more likely if fundamentals are weak. This reduces coordination failure and is in line with [24]. Knowing that the fundamentals are more probable to lie in a range suitable for a currency attack further lowers coordination failure. Likewise, the effect of greater public precision on the probability of a currency crisis is ambiguous if fundamentals are strong.

## A.2 The posterior distribution

Recall that  $\rho \sim U [0, \rho^H]$ . We are interested in the distribution of  $Z \equiv \int_0^{\rho^H} \frac{Y}{\rho^H} d\rho = u(Y)$ , where  $Y \equiv \theta_2 | \theta_1$ . It is evident that the distribution of the posterior  $Z$  is normally distributed, as we face the equivalent of a sum of independent normally distributed random variables. The moments of the posterior random variable are computed by the moment-generating function technique. The moment-generating function of  $Z = u(Y)$  is given by:

$$M_Z(t) = E[e^{tz}] = E[e^{tu(y)}] = \int_{-\infty}^{\infty} e^{tu(y)} f(y) dy,$$

where:

$$f(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{y - \mu_y}{\sigma_y} \right)^2}$$

with  $\sigma_y^2 = \frac{1 - \rho_i^2}{\alpha}$  and  $\mu_y = \rho_i \theta_1 + (1 - \rho_i) \mu$ .

Hence:

$$M_{\rho_i}(t) = \sqrt{\frac{\alpha}{2\pi(1-\rho_i^2)}} \int_{-\infty}^{\infty} e^{\left\{ ty - \frac{\alpha}{2(1-\rho_i^2)} (y - \rho_i\theta_1 - (1-\rho_i)\mu)^2 \right\}} dy,$$

which can be reformulated as:

$$M_{\rho_i}(t) = e^{\left\{ (\rho_i\theta_1 + (1-\rho_i)\mu)t + \frac{(1-\rho_i^2)}{2\alpha} t^2 \right\}}.$$

The summation is of independent and identically distributed random variables, which follow the distribution of  $Y$ , has to be correctly weighted with the relative size of the interval over which the area is computed. Let size of a step be given by  $\Delta_i$ , then its relative size is given by  $(1/\rho^H) * \Delta_i$ . For infinitesimal steps ( $\Delta_i \rightarrow 0$ ), we have an approximation of the Riemann integral.

$$M_Z(t) = e^{\left\{ \sum_{i=0}^{\infty} \left[ \frac{\Delta_i}{\rho^H} * \left( (\rho_i\theta_1 + (1-\rho_i)\mu)t + \frac{(1-\rho_i^2)}{2\alpha} t^2 \right) \right] \right\}},$$

where the exponent can be rewritten as:

$$\begin{aligned} \sum_{i=0}^{\infty} \left[ \frac{\Delta_i}{\rho^H} * \left( (\rho_i\theta_1 + (1-\rho_i)\mu)t + \frac{(1-\rho_i^2)}{2\alpha} t^2 \right) \right] &= \int_0^{\rho^H} \left( \frac{\rho_i\theta_1 + (1-\rho_i)\mu}{\rho^H} t + \frac{(1-\rho_i^2)}{2\alpha\rho^H} t^2 \right) d\rho_i \\ &= \left( \frac{\rho^H}{2} (\theta_1 - \mu) + \mu \right) t + \left( 3 - (\rho^H)^2 \right) \frac{t^2}{6\alpha}. \end{aligned}$$

The first and second derivatives read:

$$M'_Z(t) = \left( \left( \frac{\rho^H}{2} (\theta_1 - \mu) + \mu \right) + \left( 3 - (\rho^H)^2 \right) \frac{t}{3\alpha} \right) * e^{\left\{ \left( \frac{\rho^H}{2} (\theta_1 - \mu) + \mu \right) t + \left( 3 - (\rho^H)^2 \right) \frac{t^2}{6\alpha} \right\}}$$



and

$$M_Z''(t) = \frac{(3-(\rho^H)^2)}{3\alpha} * e^{\left\{\left(\frac{\rho^H}{2}(\theta_1-\mu)+\mu\right)t+(3-(\rho^H)^2)\frac{t^2}{6\alpha\rho^H}\right\}} \\ + \left(\frac{\rho^H}{2}(\theta_1-\mu)+\mu+\frac{(3-(\rho^H)^2)t}{3\alpha}\right)^2 * e^{\left\{\left(\frac{\rho^H}{2}(\theta_1-\mu)+\mu\right)t+(3-(\rho^H)^2)\frac{t^2}{6\alpha\rho^H}\right\}}$$

Hence the first two central moments are given by:

$$E[Z] = M_Z'(t)|_{t \rightarrow 0} = \frac{\rho^H}{2}(\theta_1-\mu)+\mu \quad (65)$$

$$Var[Z] = E[Z^2] - (E[Z])^2 = \frac{(3-(\rho^H)^2)}{3\alpha} + \left(\frac{\rho^H}{2}(\theta_1-\mu)+\mu\right)^2 - \left(\frac{\rho^H}{2}(\theta_1-\mu)+\mu\right)^2 \quad (66)$$

$$Var[Z] = \frac{3-(\rho^H)^2}{3\alpha} \in \left[\frac{2}{3\alpha}, \frac{1}{\alpha}\right]. \quad (67)$$

For the special case of  $\rho = 0$ , we have that  $Var[Z]|_{\rho^H \rightarrow 0} = \frac{1}{\alpha}$ .

## A.3 Region 2

### A.3.1 The special case $n = 0$

Given the normality of the posterior, the uninformed agent  $i$  uses her private signal,  $x_{i2U}$ , to update her beliefs about the fundamental of region 2:

$$\theta_2 | \theta_1, x_{i2U} \sim \mathcal{N}\left(\Theta_{i2U}, \frac{1}{\frac{\alpha}{1-\rho_H^2/3} + \gamma}\right) \quad (68)$$

$$\Theta_{i2U} \equiv \frac{\alpha[\mu - \rho_H \frac{\mu - \theta_1}{2}] + \gamma x_{i2U}(1 - \rho_H^2/3)}{\alpha + \gamma(1 - \rho_H^2/3)} \quad (69)$$

Second, the expected share of agents,  $A_{2U}$ , receiving signal lower than the uninformed agents' threshold signal (to be determined) is:

$$E[A_{2U}|\theta_1, \Theta_{2U}^*] = \Pr\{x_{j2U} < x_{2U}^*|\Theta_{2U}^*\} \quad (70)$$

$$\Rightarrow \Pr\{\Theta_{j2U} < \Theta_{2U}^*|\Theta_{2U}^*\} = \Phi\left(\sqrt{\delta_1}[\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}]\right) \quad (71)$$

$$\delta_1 \equiv \frac{\alpha^2(\alpha + \gamma(1 - \rho_H^2/3))}{\gamma(1 - \rho_H^2/3)^2(\alpha + 2\gamma(1 - \rho_H^2/3))} \quad (72)$$

This case collapses to the one studied previously in the absence of correlation ( $\rho_H = 0$ ). As before, the threshold for the equilibrium posterior mean  $\Theta_{2U}^*$  is implicitly defined by an indifference condition between attacking and not attacking:

$$\Phi\left(\sqrt{\delta_1}[\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}]\right) = \Theta_{2U}^* \quad (73)$$

The threshold for the regional fundamental can be found as:

$$\theta_{2U}^* = \Phi\left(\frac{\Theta_{2U}^* \left(\frac{\alpha + \gamma(1 - \rho_H^2/3)}{\gamma(1 - \rho_H^2/3)}\right) - \frac{\alpha}{\gamma(1 - \rho_H^2/3)}[\mu - \rho_H \frac{\mu - \theta_1}{2}] - \theta_{2U}^*}{\sqrt{1/\gamma}}\right) \quad (74)$$

Again the LHS increases in  $\theta_{2U}$ , while the RHS decreases in  $\theta_{2U}$ . There exists a unique threshold of fundamentals in the second region  $\theta_{2U}^* \in (0, 1)$ .

### A.3.2 The general case $0 < n < 1$

We analyze in turn the problem of informed and uninformed agents.

**Informed agents** Informed agent  $i$  uses her signal  $x_{i2I}$  to update her beliefs about the regional fundamental:

$$\theta_2 | \theta_1, x_{i2I} \sim \mathcal{N} \left( \Theta_{i2I}, \frac{1}{\frac{\alpha}{1-\rho^2} + \gamma} \right) \quad (75)$$

$$\Theta_{i2I} \equiv \frac{\alpha[\rho\theta_1 + (1-\rho)\mu] + \gamma(1-\rho^2)x_{i2I}}{\alpha + \gamma(1-\rho^2)} \quad (76)$$

After some derivation<sup>12</sup>, the expected incidence of informed attacking speculators evaluated at the equilibrium posterior mean  $\Theta_{2I}^*$  (to be determined) is:

$$\Pr\{\Theta_{j2I} < \Theta_{2I}^*(\rho) | \Theta_{2I}^*(\rho)\} = E[A_{2I} | \Theta_{2I}^*(\rho)] \quad (82)$$

$$= \Phi \left( \sqrt{\delta_2} [\Theta_{2I}^*(\rho) - \rho\theta_1 - (1-\rho)\mu] \right) \quad (83)$$

$$\delta_2 \equiv \frac{\alpha^2(\alpha + \gamma(1-\rho^2))}{(1-\rho^2)^2\gamma(\alpha + 2\gamma(1-\rho^2))} \quad (84)$$

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<sup>12</sup>The conditional distribution of another informed speculator  $j$ 's signal is given by:

$$x_{j2I} | \Theta_{i2I} \sim \mathcal{N} \left( \Theta_{i2I}, \frac{\alpha + 2\gamma(1-\rho^2)}{\gamma(\alpha + \gamma(1-\rho^2))} \right) \quad (77)$$

Conditional on  $\Theta_{i2I}$ , the expected share of agents that are informed and receive a signal lower than  $x_{j2I} < x_{i2I}$  is given by the probability of the event:

$$E[\mathbf{1}\{\Theta_{j2I} < \Theta_{i2I} | \Theta_{i2I}\}] = \Pr\{\Theta_{j2I} < \Theta_{i2I} | \Theta_{i2I}\} \quad (78)$$

We can use equation (76) to rewrite the inequality as:

$$\Theta_{j2I} < \Theta_{i2I} \Leftrightarrow \frac{\alpha[\rho\theta_1 + (1-\rho)\mu] + \gamma(1-\rho^2)x_{j2I}}{\alpha + \gamma(1-\rho^2)} < \Theta_{i2I} \quad (79)$$

$$\Leftrightarrow x_{j2I} < \frac{\alpha + \gamma(1-\rho^2)}{\gamma(1-\rho^2)} \Theta_{i2I} - \frac{\alpha}{\gamma(1-\rho^2)} [\rho\theta_1 + (1-\rho)\mu] \quad (80)$$

As a result, we arrive at:

$$\begin{aligned} \Pr\{\Theta_{j2I} < \Theta_{i2I} | \Theta_{i2I}\} &= \Pr\left\{x_{j2I} < \frac{\alpha + \gamma(1-\rho^2)}{\gamma(1-\rho^2)} \Theta_{i2I} - \frac{\alpha}{\gamma(1-\rho^2)} [\rho\theta_1 + (1-\rho)\mu] \mid \Theta_{i2I}\right\} \\ &= \Phi \left( \sqrt{\delta_3} [\Theta_{i2I} - \rho\theta_1 - (1-\rho)\mu] \right) \end{aligned} \quad (81)$$

An informed speculator  $i$  is indifferent between attacking and not attacking at the threshold posterior mean  $\Theta_{i2I} = \Theta_{2I}^*(\rho)$ :<sup>13</sup>

$$\Theta_{2I}^*(\rho; \Theta_{2U}^*) = n * \Phi \left( \sqrt{\delta_2} [\Theta_{2I}^*(\rho) - \rho\theta_1 - (1 - \rho)\mu] \right) + (1 - n)E[A_{2U}|\Theta_{2I}^*(\rho)] \quad (85)$$

where  $E[A_{2U}|\Theta_{2I}^*(\rho)]$  is to be determined, taking  $\Theta_{2U}^*$  as given. We now determine  $E[A_{2U}|\Theta_{2I}^*(\rho)]$ , where  $\Theta_{2I}^*(\rho)$  is still to be determined:<sup>14</sup>

$$E[A_{2U}|\Theta_{2I}^*(\rho)] = \Phi \left( \sqrt{\delta_3} \left[ \frac{\alpha}{\gamma(1 - \rho_H^2/3)} (\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}) + \Theta_{2U}^* - \Theta_{2I}^*(\rho) \right] \right) \quad (88)$$

Notice that  $\delta_3 \equiv (\frac{\gamma}{\alpha}(1 - \rho^2))^2 \delta_2$  and, hence,  $E[A_{2U}|\Theta_{2I}^*(\rho)]$  is a function of  $\rho$ . Equation (85) can be rewritten such that it implicitly defines  $\Theta_{2I}^*(\rho)$  for any  $\rho$ , taking  $\Theta_{2U}^*$  as given:

$$\begin{aligned} \Theta_{2I}^*(\rho) &= n\Phi \left( \sqrt{\delta_2} [\Theta_{2I}^*(\rho) - \rho\theta_1 - (1 - \rho)\mu] \right) \\ &+ (1 - n)\Phi \left( \sqrt{\delta_3} \left[ \frac{\alpha}{\gamma(1 - \rho_H^2/3)} (\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}) + \Theta_{2U}^* - \Theta_{2I}^*(\rho) \right] \right) \end{aligned} \quad (89)$$

Before demonstrating existence and uniqueness of an equilibrium in thresh-

<sup>13</sup> $E[u(a_{i2I} = 1, A_{2I}, A_{2U}, \theta_2)|\Theta_{2I}^*(\theta_1, \rho, \lambda_U)] = E[n * A_{2I} + (1 - n) * A_{2U} - \theta_2|\Theta_{2I}^*]$

<sup>14</sup>Recall the mapping between  $\Theta_{i2U}$  and  $x_{i2U}$  from equation (69). Given the information on the correlation, informed speculators compute how many uninformed speculators can be expected to receive a signal below  $x_{2U}^*$ :

$$\begin{aligned} E[A_{2U}|\Theta_{i2I}(\rho)] &= \Pr\{x_{j2U} < \frac{\alpha + \gamma(1 - \rho_H^2/3)}{\gamma(1 - \rho_H^2/3)} \Theta_{i2U} - \frac{\alpha}{\gamma(1 - \rho_H^2/3)} [\mu - \rho_H \frac{\mu - \theta_1}{2}] | \Theta_{i2I}(\rho)\} \\ &= \Phi \left( \frac{\frac{\alpha + \gamma(1 - \rho_H^2/3)}{\gamma(1 - \rho_H^2/3)} \Theta_{i2U} - \frac{\alpha}{\gamma(1 - \rho_H^2/3)} [\mu - \rho_H \frac{\mu - \theta_1}{2}] - \Theta_{i2I}(\rho)}{\sqrt{\text{Var}[x_{j2U}|\Theta_{i2I}(\rho)]}} \right) \\ &= \Phi \left( \sqrt{\delta_3} \left[ \frac{\alpha}{\gamma(1 - \rho_H^2/3)} (\Theta_{i2U} - \mu + \rho_H \frac{\mu - \theta_1}{2}) + \Theta_{i2U} - \Theta_{i2I}(\rho) \right] \right) \end{aligned} \quad (86)$$

where:

$$\delta_3 \equiv \frac{\gamma(\alpha + \gamma(1 - \rho^2))}{\alpha + 2\gamma(1 - \rho^2)} \quad (87)$$

old strategies, it shows to be insightful to examine the partial derivatives and to derive some important preliminary results. First, we find the set of partial derivatives of  $\Theta_{2I}^*(\rho)$ , for any  $\rho \in [0, \rho_H]$  as:

$$\frac{d\Theta_{2I}^*(\rho)}{d\Theta_{2U}^*} = \frac{(1-n)\sqrt{\delta_3} \frac{\alpha + \gamma(1-\rho_H^2/3)}{\gamma(1-\rho_H^2/3)} \phi_{IU}}{1 - n\sqrt{\delta_2}\phi_{II} + (1-n)\sqrt{\delta_3}\phi_{IU}} \quad (90)$$

$$\phi_{II} \equiv \sqrt{\delta_2}[\Theta_{2I}^*(\rho) - \rho\theta_1 - (1-\rho)\mu] \quad (91)$$

$$\phi_{IU} \equiv \sqrt{\delta_3} \left[ \frac{\alpha}{\gamma(1-\rho_H^2/3)} (\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}) + \Theta_{2U}^* - \Theta_{2I}^*(\rho) \right] \quad (92)$$

Notice that  $d\Theta_{2I}^*/d\Theta_{2U}^* \in (0, 1)$  for all  $\rho$  if the private signal is sufficiently precise relative to the public signal.<sup>15</sup>

**Uninformed agents** Next, we turn to uninformed speculators. As demonstrated in section 3.2.1, the expected share of uninformed agents,  $A_{2U}$ , conditional on  $\Theta_{2U}^*$  who receive a signal lower than the uninformed agents' threshold signal, which still needs to be determined, is:

$$E[A_{2U}|\Theta_{2U}^*] = \Phi \left( \sqrt{\delta_1} [\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}] \right) \quad (93)$$

The indifference condition has to hold with equality for uninformed speculator  $i$  with the threshold posterior mean  $\Theta_{i2U} = \Theta_{2U}^*$ :

$$\Theta_{2U}^* = nE[A_{2I}|\Theta_{2U}^*] + (1-n)E[A_{2U}|\Theta_{2U}^*] \quad (94)$$

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<sup>15</sup>More precisely, all we require is  $\gamma > \underline{\gamma} < \infty$ . To see this, note that the numerator is always positive and that the denominator is positive if  $\delta_2 < 2\pi$ . A sufficient condition for the fraction to be less than one is  $1 > n\sqrt{\delta_2}\phi_{II} + (1-n)\sqrt{\delta_3}\phi_{IU} \frac{1-\rho^2}{1-\rho_H^2/3}$ , which is always satisfied if  $\delta_2$  is sufficiently small or, equivalently,  $\gamma$  is sufficiently high. Note that  $\delta_2 < 2\pi$  is insufficient.

where  $E[A_{2I}|\Theta_{2U}^*] = \int_0^{\rho^H} \frac{E[A_{2I}(\tilde{\rho})|\Theta_{2U}^*]}{\rho_H} d\tilde{\rho}$  also takes expectation over the possible realisations of the correlation between regional fundamentals and (the sequence of)  $\Theta_{2I}^*(\rho)$ , implicit in  $A_{2I}(\rho)$ , is taken as given. Combining equations (93) and (94):

$$\Theta_{2U}^* = n \int_0^{\rho^H} \frac{E[A_{2I}(\tilde{\rho})|\Theta_{2U}^*]}{\rho_H} d\tilde{\rho} + (1-n)\Phi\left(\sqrt{\delta_1}[\Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2}]\right) \quad (95)$$

We determine  $E[A_{2I}(\rho)|\Theta_{2U}^*]$  for any  $\rho$  similar to the informed case.<sup>16</sup>

$$\begin{aligned} E[A_{2I}^*(\rho)|\Theta_{2U}^*] &= \Phi\left(\sqrt{\delta_4}\left[\frac{\alpha}{\gamma(1-\rho^2)}(\Theta_{2I}^*(\rho) - \rho\theta_1 - (1-\rho)\mu) + \Theta_{2I}^*(\rho) - \Theta_{2U}^*\right]\right) \\ \delta_4 &= \left(\frac{\gamma}{\alpha}(1 - \rho_H^2/3)\right)^2 \delta_1 \end{aligned} \quad (100)$$

Inserting into equation (95), the equilibrium condition for  $\Theta_{2U}^*$ , taking

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<sup>16</sup>Taking the perspective of uninformed agent  $i$ , the distribution of another informed speculator  $j$ 's signal conditional on  $\Theta_{i2U}$  is given by:

$$x_{j2I}|\Theta_{i2U} \sim \mathcal{N}\left(\Theta_{i2U}, \frac{\alpha + 2\gamma(1 - \rho_H^2/3)}{\gamma(\alpha + \gamma(1 - \rho_H^2/3))}\right) \quad (96)$$

Conditional on  $\Theta_{i2U}$ , the expected share of agents that are informed and receive a signal lower than  $x_{j2I} < x_{2I}^*$  is given by the probability of the event:

$$\Pr\{\Theta_{j2I} < \Theta_{2I}^*|\Theta_{i2U}\} = \Phi\left(\sqrt{\delta_4}\left[\frac{\alpha}{\gamma(1-\rho^2)}(\Theta_{2I}^* - \rho\theta_1 - (1-\rho)\mu) + \Theta_{2I}^* - \Theta_{i2U}\right]\right) \quad (97)$$

with:

$$\delta_4 = \frac{\gamma(\alpha + \gamma(1 - \rho_H^2/3))}{\alpha + 2\gamma(1 - \rho_H^2/3)} \quad (98)$$

Evaluated at the equilibrium posterior means (to be determined), we have reach the result.

the sequence of  $\{\Theta_{2I}^*(\rho)\}$  as given, is:

$$\Theta_{2U}^* = n \int_0^{\rho_H} \frac{\Phi \left( \sqrt{\delta_4} \left[ \frac{\alpha}{\gamma(1-\tilde{\rho}^2)} (\Theta_{2I}^*(\tilde{\rho}) - \tilde{\rho}\theta_1 - (1-\tilde{\rho})\mu) + \Theta_{2I}^*(\tilde{\rho}) - \Theta_{2U}^* \right] \right)}{\rho_H} d\tilde{\rho} + (1-n) \Phi \left( \sqrt{\delta_1} \left[ \Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2} \right] \right) \quad (101)$$

Second, we find the set of partial derivatives of  $\Theta_{2I}^*$ , for any  $\rho \in [0, \rho_H]$  as:

$$\frac{d\Theta_{2U}^*}{d\Theta_{2I}^*(\rho)} = \frac{\frac{n\sqrt{\delta_4}}{\rho_H} \frac{\alpha + \gamma(1-\rho^2)}{\gamma(1-\rho^2)} \phi_{UI}(\rho)}{1 + \frac{n\sqrt{\delta_4}}{\rho_H} \int_0^{\rho_H} \phi_{UI}(\tilde{\rho}) d\tilde{\rho} - (1-n)\sqrt{\delta_1}\phi_{UU}} \quad (102)$$

$$\phi_{UI} \equiv \sqrt{\delta_4} \left[ \frac{\alpha}{\gamma(1-\rho^2)} (\Theta_{2I}^*(\rho) - \rho\theta_1 - (1-\rho)\mu) + \Theta_{2I}^*(\rho) - \Theta_{2U}^* \right] \quad (103)$$

$$\phi_{UU} \equiv \sqrt{\delta_1} \left[ \Theta_{2U}^* - \mu + \rho_H \frac{\mu - \theta_1}{2} \right] \quad (104)$$

As before,  $\frac{d\Theta_{2U}^*}{d\Theta_{2I}^*(\rho)} \in (0, 1)$  for all  $\rho$  if the private signal is sufficiently precise relative to the public signal.<sup>17</sup>

**Threshold of the regional fundamental** Finally, we obtain the threshold of the regional fundamental  $\theta_2^*$  below which there is a currency attack ( $\theta_2 < \theta_2^*$ ). For a realised fundamental  $\theta_2$  and correlation  $\rho$ :

$$x_{i2} \sim \mathcal{N} \left( \theta_2, \frac{1}{\gamma} \right) \quad (105)$$

$$\Theta_{i2I} \sim \mathcal{N} \left( \frac{\alpha[\rho\theta_1 + (1-\rho)\mu] + \gamma(1-\rho^2)\theta_2}{\alpha + \gamma(1-\rho^2)}, \frac{(\gamma(1-\rho^2))^2}{\gamma(\alpha + \gamma(1-\rho^2))^2} \right) \quad (106)$$

$$\Theta_{i2U} \sim \mathcal{N} \left( \frac{\alpha[\mu - \rho_H \frac{\mu - \theta_1}{2}] + \gamma(1-\rho_H^2/3)\theta_2}{\alpha + \gamma(1-\rho_H^2/3)}, \frac{(\gamma(1-\rho_H^2/3))^2}{\gamma(\alpha + \gamma(1-\rho_H^2/3))^2} \right) \quad (107)$$

<sup>17</sup>More precisely, all we require is  $\gamma > \underline{\gamma}_1 < \infty$ . To see this, note that the numerator is always positive and that the denominator is positive if  $\delta_1 < 2\pi$ . A sufficient condition for the fraction to be less than one is  $1 > \frac{n}{\rho_H} \frac{1-\rho_H^2/3}{1-\rho^2} \sqrt{\delta_1} \phi_{UI} + (1-n)\sqrt{\delta_1}\phi_{UU}$ , which is always satisfied if  $\delta_1$  is sufficiently small or, equivalently,  $\gamma$  is sufficiently high. Note that  $\delta_1 < 2\pi$  is insufficient.

In equilibrium:

$$\begin{aligned}
\theta_2^* &= nA_{2I}^* + (1-n)A_{2U}^* = n \Pr\{\Theta_{i2I} < \Theta_{2I}^*(\rho)\} + (1-n) \Pr\{\Theta_{i2U} < \Theta_{2U}^*\} \\
\theta_2^* &= n\Phi\left(\frac{\frac{\alpha+\gamma(1-\rho^2)}{\gamma(1-\rho^2)}\Theta_{2I}^*(\rho) - \frac{\alpha[\rho\theta_1+(1-\rho)\mu]}{\gamma(1-\rho^2)} - \theta_2^*}{\sqrt{1/\gamma}}\right) \\
&\quad + (1-n)\Phi\left(\frac{\frac{\alpha+\gamma(1-\rho_H^2/3)}{\gamma(1-\rho_H^2/3)}\Theta_{2U}^* - \frac{\alpha[\mu-\rho_H\frac{\mu-\theta_1}{2}]}{\gamma(1-\rho_H^2/3)} - \theta_2^*}{\sqrt{1/\gamma}}\right)
\end{aligned} \tag{109}$$

#### A.4 Proof of Proposition 1

The proof proceeds in two steps. The first step shows that there is a unique solution  $(\{\Theta_{2I}^*(\rho)\}, \Theta_{2U}^*)$  to the sequence of equilibrium conditions. The second step demonstrates that an individual speculator never finds it individually profitable to deviate from playing a threshold strategy. Each step needs to be performed for informed and uninformed speculators.

*Step 1:* Start with uninformed agents whose equilibrium condition is given by equation (15). The slope of the left-hand side is one, while the slope of the right-hand side is strictly smaller than one if the private signal is sufficiently precise. Then, there exists a unique solution  $\Theta_{2U}^*$  for any sequence of thresholds  $\{\Theta_{2I}^*(\rho)\}$ . To see this, take the derivative of the right-hand side:

$$\begin{aligned}
\frac{dRHS}{d\Theta_{2U}^*} &= \frac{n}{\rho_H} \int_0^{\rho_H} \left( \sqrt{\delta_4} \left[ \frac{\alpha + \gamma(1-\tilde{\rho}^2)}{\gamma(1-\tilde{\rho}^2)} \frac{d\Theta_{2I}^*(\tilde{\rho})}{d\Theta_{2U}^*} - 1 \right] \right) \phi_{UI}(\tilde{\rho}) d\tilde{\rho} \\
&\quad + (1-n) \sqrt{\delta_1} \phi_{UU}
\end{aligned} \tag{110}$$

The derivate is positive. Inserting  $\frac{d\Theta_{2I}^*(\rho)}{d\Theta_{2U}^*} = 1$  for all  $\rho \in [0, \rho_H]$  yields the



following sufficient condition:

$$1 \geq \sqrt{\delta_1} \left[ (1-n)\phi_{UU} + \frac{n}{\rho_H} \int_0^{\rho_H} \frac{1-\rho_H^2/3}{1-\tilde{\rho}^2} \phi_{UI}(\tilde{\rho}) d\tilde{\rho} \right] \quad (111)$$

Given that  $\phi \leq \frac{1}{2\pi}$ , a sufficiently precise private signal ensures this condition.

Likewise, we consider the (sequence of) informed agents with the associated equilibrium condition (14). The slope of the left-hand side is again unity, while the slope of the right-hand side is guaranteed to be smaller than one if the private signal is sufficiently precise. This ensures that there is a unique threshold  $\Theta_{2I}^*(\rho)$  for any  $\Theta_{2U}^*$ . To see this, the slope of the right-hand side is:

$$\begin{aligned} \frac{dRHS}{d\Theta_{2I}^*(\rho)} &= n\sqrt{\delta_2}\phi_{II} \quad (112) \\ &+ (1-n) \left( \sqrt{\delta_3} \left[ \frac{\alpha + \gamma(1-\rho_H^2/3)}{\gamma(1-\rho_H^2/3)} \frac{d\Theta_{2U}^*}{d\Theta_{2I}^*(\rho)} - 1 \right] \right) \phi_{IU} \quad (113) \end{aligned}$$

As in the previous case, we find a simple sequence of sufficient conditions:

$$1 \geq \sqrt{\delta_2} \left[ n\phi_{II} + (1-n)\phi_{IU}(\rho) \frac{1-\rho^2}{1-\rho_H^2/3} \right] \quad (114)$$

Again, each of these conditions holds for a sufficiently high  $\gamma$ .

*Step 2:* The proof is completed by showing that there is no privately optimal deviation from the prescribed threshold strategy. This follows Ahnert and Nelson [2], which is an extension of the proof in Morris and Shin [18].

Consider uninformed speculators first. Upon the receipt of the private

signal  $x_{i2U}$ , they form the posterior about the regional fundamental:

$$\Theta_{i2U} = \frac{\alpha[(1 - 0.5\rho_H)\mu + 0.5\rho_H\theta_1] + \gamma(1 - \rho_H^2/3)x_{i2U}}{\alpha + \gamma(1 - \rho_H^2/3)} \quad (115)$$

Given that everybody else uses the threshold strategy with thresholds  $\Theta_{2U}^*$  if uninformed and  $\Theta_{2I}^*(\rho)$  if informed and observing  $\rho$ , the expected fraction of attacking uninformed and informed speculators conditional on forming the posterior  $\Theta_{i2U}$  can be found. The conditional distribution of another uninformed speculator is:

$$x_{j2U}|\Theta_{i2U} \sim \mathcal{N}\left(\Theta_{i2U}, \left(\frac{\alpha + 2\gamma(1 - \rho_H^2/3)}{\gamma[\alpha + \gamma(1 - \rho_H^2/3)]}\right)\right) \quad (116)$$

such that the conditional expected fraction of attacking uninformed speculators is:

$$\begin{aligned} E[A_{2U}|\Theta_{i2U}] &= \Pr\{x_{j2U} < x_{2U}^*|\Theta_{i2U}\} \\ &= \Phi\left(\sqrt{\delta_4}[\Theta_{2U}^* - \Theta_{i2U}] + \sqrt{\delta_1}[\Theta_{2U}^* - 0.5\rho_H\theta_1 - (1 - 0.5\rho_H)\mu]\right) \end{aligned} \quad (117)$$

while the conditional expected fraction of attacking informed speculators with information  $\rho$  is:

$$E[A_{2I}(\rho)|\Theta_{i2U}] = \Phi\left(\sqrt{\delta_4}\left[\Theta_{2I}^*(\rho) - \Theta_{i2U} + \frac{\alpha}{\gamma(1 - \rho^2)}(\Theta_{2I}^*(\rho) - \rho\theta_1 - (1 - \rho)\mu)\right]\right) \quad (118)$$

for any  $\rho \in [0, \rho_H]$ . The expected utility of an attacking uninformed speculator is thus:

$$\Pi_U \equiv (1 - n)E[A_{2U}|\Theta_{i2U}] + n \int_0^{\rho_H} E[A_{2I}(\tilde{\rho})|\Theta_{i2U}]\frac{d\tilde{\rho}}{\rho_H} - \Theta_{i2U} \quad (119)$$

The partial derivatives of the payoff to an uninformed speculator are:

$$\frac{d\Pi_U}{d\Theta_{i2U}} = -1 - \sqrt{\delta_4} \left[ (1-n)\phi_{i,UU} + n \int_0^{\rho_H} \phi_{i,UI}(\tilde{\rho}) \frac{d\tilde{\rho}}{\rho_H} \right] < 0 \quad (120)$$

$$\frac{d\Pi_U}{d\Theta_{2U}^*} = (\sqrt{\delta_1} + \sqrt{\delta_4})(1-n)\phi_{i,UU} > 0 \quad (121)$$

$$\frac{d\Pi_U}{d\Theta_{2I}^*(\rho)} = n \frac{1}{\rho_H} \phi_{i,UI}(\rho) \sqrt{\delta_4} \left[ 1 + \frac{\alpha}{\gamma(1-\rho^2)} \right] > 0 \quad (122)$$

for any  $\rho \in [0, \rho_H]$  and where  $\phi_{i,UU}, \dots, \phi_{i,II}$  are defined in analogy to  $\phi_{UU}, \dots, \phi_{II}$ . Following the argument in Ahnert and Nelson [2], there exists a unique strategy that survives the infinite iteration of deletion of strictly dominant strategies for uninformed speculators: it prescribes to attack if and only if  $\Theta_{i2U} < \Theta_{2U}^*$ , which is the equilibrium threshold strategy. In other words, the best response to everyone is playing the threshold strategy is playing the threshold strategy as well if you are uninformed. The same argument holds for informed speculators as we see now.

An informed speculator who receives the private signal  $x_{i2I}$  and learned  $\rho$  forms the posterior about the regional fundamental:

$$\Theta_{i2I} = x_{i2I} + \frac{\alpha}{\gamma(1-\rho^2)} [\Theta_{i2I} - \rho\theta_1 - (1-\rho)\mu] \quad (123)$$

Given that everybody else uses the threshold strategy with thresholds  $\Theta_{2U}^*$  if uninformed and  $\Theta_{2I}^*(\rho)$  if informed and observing  $\rho$ , the expected fraction of attacking uninformed and informed speculators conditional on forming the posterior  $\Theta_{i2I}$  can be found. The conditional distribution of another informed speculator is:

$$x_{j2I} | \Theta_{i2I} \sim \mathcal{N} \left( \Theta_{i2I}, \left( \frac{\alpha + 2\gamma(1-\rho^2)}{\gamma[\alpha + \gamma(1-\rho^2)]} \right) \right) \quad (124)$$

such that the conditional expected fraction of attacking informed speculators is:

$$\begin{aligned} E[A_{2I}|\Theta_{i2I}] &= \Pr\{x_{j2I} < x_{2I}^*|\Theta_{i2I}\} \\ &= \Phi\left(\sqrt{\delta_3}[\Theta_{2I}^* - \Theta_{i2I}] + \sqrt{\delta_2}[\Theta_{2I}^* - \rho\theta_1 - (1 - \rho)\mu]\right) \end{aligned} \quad (125)$$

while the conditional expected fraction of attacking uninformed speculators is:

$$E[A_{2U}|\Theta_{i2I}] = \Phi\left(\sqrt{\delta_3}\left[\Theta_{2U}^* - \Theta_{i2I} + \frac{\alpha}{\gamma(1 - \rho_H^2/3)}(\Theta_{2U}^* - 0.5\rho_H\theta_1 - (1 - 0.5\rho_H)\mu)\right]\right) \quad (126)$$

for any  $\rho \in [0, \rho_H]$ . The expected utility of an attacking informed speculator is thus:

$$\Pi_I \equiv (1 - n)E[A_{2U}|\Theta_{i2I}] + nE[A_{2I}|\Theta_{i2I}] - \Theta_{i2I} \quad (127)$$

The partial derivatives of the payoff to an uninformed speculator are:

$$\frac{d\Pi_I}{d\Theta_{i2I}} = -1 - \sqrt{\delta_3}[(1 - n)\phi_{i,IU} + n\phi_{i,II}] < 0 \quad (128)$$

$$\frac{d\Pi_I}{d\Theta_{2I}^*} = n(\sqrt{\delta_2} + \sqrt{\delta_3})\phi_{i,II} > 0 \quad (129)$$

$$\frac{d\Pi_I}{d\Theta_{2U}^*} = (1 - n)\phi_{i,IU}\sqrt{\delta_3}\left[1 + \frac{\alpha}{\gamma(1 - \rho_H^2/3)}\right] > 0 \quad (130)$$

where the same argument as for uninformed speculators guarantees that the best response for informed agents is also to play the equilibrium threshold strategy.

## A.5 Proof of Lemma 3

Before proving the results of Lemma 3 (a) and (b) it is useful to state two preliminary results. :

$$\frac{d\Theta_{2I}^*(\rho)}{d\rho}\Big|_{\rho \rightarrow 0} \begin{cases} > 0 & \text{if } n \rightarrow 1 \\ > 0 & \text{if } n \rightarrow 0 \end{cases} \quad (131)$$

and:

$$\frac{d\left(\frac{d\Theta_{2I}^*(\rho)}{d\rho}\Big|_{\rho \rightarrow 0}\right)}{dn}\Big|_{n=0} > 0 \quad (132)$$

### A.5.1 Lemma 3 (a)

The proof is by contradiction. Consider an individual speculator who observes that all other speculators are uninformed ( $n = 0$ ). Suppose that she uses the critical thresholds of uninformed speculators,  $x_{2I}^*(\rho, n = 0) = x_{2U}^*(n = 0)$  for all  $\rho \in [0, \rho_H]$ . Then, she obtains the same expected payoff, net of the cost  $c$ , as an uninformed speculator. The objective is to show that an informed speculator strictly prefers to adjust her threshold at least for some values of  $\rho$  and that this adjustment yields a strictly higher expected payoff.

Recall that  $\theta_{2U}^*(n = 0) < \mu$  for strong fundamentals<sup>18</sup> and consider an informed speculator who observes  $\rho = 0$ . Suppose that she uses the critical threshold  $x_{2I}^*(\rho, n = 0) = x_{2U}^*(n = 0)$ . Next consider an informed speculators who observes  $\rho = \xi$ , where  $\xi > 0$  is small. Given equations

<sup>18</sup>Strong fundamentals imply that  $\theta_{2U}^*(n = 0) < (1 - \rho_H/2)\mu + \rho_H/2\theta_1$ . As we consider the crisis range of fundamentals in the first region ( $\theta_1 < \theta_1^* < \mu$ ), the stated inequality follows.

(131) and (132) and the one-to-one mapping between the threshold signal and the threshold posterior mean, the speculator uses a critical threshold  $x_{2I}^*(\xi, n = 0) > x_{2I}^*(0, n = 0)$ , leading to a contradiction. As a result, the informed speculator gains from adjusting her threshold at least for some values of  $\rho$ . This is because an adjustment in the critical threshold always strictly changes the expected payoff. (We established this result earlier when inspecting the indifference equilibrium condition.)

Since these changes in the expected payoff are strictly positive and the range of  $\rho$  for which this occurs has positive mass as  $\xi > 0$ , there always exists a sufficiently small cost  $c$  such that an individual speculator finds it optimal to acquire information. The sufficient condition given in Lemma 3 (a) can be derived following this argument.

### **A.5.2 Lemma 3 (b)**

The result can be proven by an analog argument using the first result in equation (131).

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