

**Vortrag im**  
**Volkswirtschaftlichen Kolloquium**

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**“Equilibria in discrete and continuous  
second price all-pay auctions,  
convergence of yoyo phenomena”**

Montag, 08. Oktober 2018  
18.00 Uhr, Mathematikgebäude, SR 127

**Interessierte sind herzlich eingeladen!**

**Abstract:**

The paper is about mixed strategy Nash equilibria in discrete second price all-pay auctions with a limit budget. Two players fight over a prize of value  $V$ . Each player submits a bid lower or equal to  $M$ , the limit budget. The prize goes to the highest bidder but both bidders pay the lowest bid.  $V$ ,  $M$  and the bids are integers. The paper studies the convergence of the mixed Nash equilibrium probability distribution in the discrete auction to the mixed Nash equilibrium probability distribution in the more well-known continuous second price all-pay auction -or static war of attrition.

We establish that the- expected- convergence between discrete and continuous equilibrium distributions is in no way automatic. Both distributions converge for  $V$  odd and large, but, for even values of  $V$ , the discrete distribution is quite strange and obeys a singular yoyo phenomenon: the probabilities assigned to two adjacent bids are quite different, one probability being much lower than the continuous one, the adjacent probability being much larger. So the discrete probabilities, for  $V$  even, don't converge to the continuous ones. Yet there is a convergence, when turning to sums: the sums of the discrete probabilities of two adjacent bids converge to the sums of the continuous probabilities of the same two bids for large values of  $V$ . It is shown in the paper that the yoyo phenomenon doesn't disappear - it is even strengthened- when switching to lower natural bid increments, like 0.5 or 0.1. More generally, it is shown that convergence is an exception rather than the rule and that it requires a special link between  $V$ ,  $M$  and the bid increment. It follows a lack of continuity between the discrete Nash equilibria and the continuous Nash equilibria.