The Optimality of Heterogeneous Tournaments

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Abstract

We investigate the effect of employee heterogeneity on the incentive to put forth effort in a market-based tournament. Employers use the tournament's outcome to estimate employees' abilities and accordingly condition their wage offers. Employees put forth effort, because by doing so they increase the probability of outperforming the rival, thereby increasing their ability assessment and thus the wage offer. We demonstrate that the tournament outcome provides more information about employees' abilities in case they are heterogeneous. Thus, employees get a higher incentive to affect the tournament outcome, and employers find it optimal to hire heterogeneous contestants.

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1 Introduction

In many firms, promotions form an important incentive. Employees put forth effort to perform better than their colleagues and, thus, to be considered for promotion and concomitantly offered an increase in compensation. The current paper analyzes how heterogeneity among the employees affects the incentive to exert effort in a promotion tournament. A rationale for designing heterogeneous promotion tournaments is provided, based on what we believe is an important learning effect that the tournament literature has overlooked.

The seminal paper on promotion tournaments is the one by Lazear and Rosen (1981). They consider a situation in which two employees compete for a promotion. A key feature of their model is that the employer commits to pay wages (or prizes) to both the promoted and the non-promoted employee before the tournament starts. Lazear and Rosen find that heterogeneity among the employees is detrimental from an incentive perspective (unless the employer introduces handicaps to counteract the heterogeneity). This is because heterogeneity lowers the marginal effect of a player's effort on his probability of being promoted, i.e. of producing higher output than does the opponent. Intuitively, the employee of lower ability realizes that he is unlikely to overcome the ability advantage of the opponent and reduces his effort. Then, of course, the employee of higher ability can afford to relax and reduce his effort as well.[1]

Recently, however, the assumption that the employer can commit to pay different prizes at the beginning of the tournament has come under scrutiny. Following Waldman (1984), some tournament papers appeared that restrict the power of the employer to commit to a certain set of prizes.[2] Instead, they argue that post-tournament wages are determined by a bidding process, taking into account the tournament outcome. In particular, the labor market (i.e. firms other than the current employer) understands promotion as a (positive) signal of employee's ability. Accordingly, promotion induces the labor market to upgrade the assessment of the employee's ability, which consequently leads to higher wage offers for the employee. Because of this, employees have an incentive to vie for promotion.

In such a market-based tournament, heterogeneity of employees affects the employees' payoff function in two ways. On the one hand, the effect is the same as that in the model

[1] See Gürtler and Kräkel (2010).

[2] See Zábojník and Bernhardt (2001), Ghosh and Waldman (2010), DeVaro (2011), DeVaro and Waldman (2012), Zábojník (2012), Waldman (forthcoming).

by Lazear and Rosen: heterogeneity lowers the marginal effect of effort on the probability of being promoted. On the other hand, heterogeneity has an effect on how the labor market uses the tournament outcome to update the assessment of the employees' abilities. It is found that, in certain situations (i.e. for certain families of ability distributions) the labor market learns more from the tournament outcome about the employees' abilities, when they are heterogeneous than when they are homogeneous. This observation is based on the following intuition: In a promotion tournament with employees who are reckoned to be similar in terms of ability, the tournament outcome is attributed mainly to luck. Therefore, the ability assessment of both the promoted and the non-promoted employee does not change much. On the contrary, if an employee who is reckoned to be of low ability, performs better than does a high-ability employee, his ability assessment is substantially upgraded. Similarly, the employee who is reckoned to be of high ability suffers significant downgrading of his ability assessment when he loses against a low-ability employee. The significant change in ability assessment, in turn, leads to a strong change in the wage offered to the employee. In a heterogeneous tournament, in which ability assessments, and thus wages, are more sensitive to the tournament outcome, employees have a higher incentive to win the tournament. This effect may be so strong that it more than compensates the original effect identified by Lazear and Rosen, thus making it optimal for the employer to hire heterogeneous contestants.

Several empirical studies exist that provide support for our results. DeVaro and Waldman (2012) analyze data from a medium-sized US firm in the financial-services industry. They find that, upon promotion, employees with a Masters degree or a Ph.D. receive a smaller wage increase as compared to employees with a Bachelors degree. This observation is consistent with the present result that, following a promotion, the ability assessment of low-ability employees is upgraded to a stronger degree than that of high-ability employees. Bognanno and Melero (2012), using data from the British Household Panel Survey, obtain similar results: less educated employees receive larger wage increases upon promotion. DeVaro (2006) provides some indirect support to the model being presented here. Analyzing promotion decisions, using data from the Multi-City Study of Urban Inequality, he finds that employers react to factors that suppress incentives by countering and increasing the spread between winner and loser prizes. This finding is in line with the results of this paper. As indicated, heterogeneity among employees reduces the marginal effect of effort on the probability of being promoted. This induces employees to reduce their effort. However, as argued before, it is possible that the labor market changes the assessment of the employees' abilities to a stronger degree after observing the tournament outcome. As a consequence, the difference between the wage offers from outside employers to the promoted and non-promoted employee increases. If the current employer matches these outside offers, he tends to react to higher heterogeneity of the employees by increasing the spread between prizes. Finally, of course, our paper can explain why recent studies observe benefits from designing heterogeneous workgroups (e.g., Hamilton et al. 2003, Franck and Nüesch 2010).

In addition to the literature cited so far, the paper is related to the literature on learning in tournaments. This literature focuses on the question of whether tournaments succeed at identifying (and selecting) the most able contestant, i.e. whether the most able contestant wins the tournament. Meyer (1991), for example, considers a series of tournaments between two heterogeneous employees. She demonstrates that selection efficiency can be improved by biasing the tournament results. If only ordinal information about the employees' performances is available, optimal bias (which, in most cases, favors the actual leader in the tournament) will increase the tournament's information content such that the information becomes a sufficient statistic for cardinal information. Clark and Riis (2001) show that efficient selection can be achieved by combining a promotion tournament with absolute performance standards. In their model, there are three tournament prizes, and the tournament's winner receives the highest prize only if his performance additionally surpasses a threshold level. By using the performance standards, the employer receives further information about the employees' abilities that he can use to select employees efficiently. Hvide and Kristiansen (2003) emphasize the relevance of the selection problem. They examine a promotion tournament, in which the employees can choose strategies of different risks. They find that selection efficiency may be low, because low-ability employees might choose risky strategies and therefore may overturn their ability disadvantage. Finally, Chen (2003) and Münster (2007) allow for sabotage in tournaments. They find that high-ability employees are strongly sabotaged, which in turn leads to low selection efficiency.[3] One needs to note that, in all these papers, the employer can either commit to certain tournament prizes at the beginning of the tournament or prizes are exogenously given. This means that none of the papers assumes that prizes are offered on the basis of a bidding process in the labor market. Obviously, this approach contrasts with that of the current paper, in which employer's inability to commit to a set of prizes

[3] See also Gürtler and Münster (2010) and Gürtler et al. (forthcoming).

at the beginning of the tournament is a necessary condition for the optimality of employee heterogeneity.

Finally, the current analysis is related to Rantakari (2012) who considers a situation, in which each employee puts forth two-dimensional effort to affect the value of a project for the employer as also for himself. The project that yields a higher value for the employer is realized by the employer. Accordingly, employees engage themselves in a kind of competition against each other, because they prefer their own project to be realized. Rantakari demonstrates that the employer may benefit from hiring heterogeneous employees. This finding depends on "winner prizes" being endogenous in the sense that they depend on the employees' efforts. This is similar to the contention of the current paper, in which the benefit that accrues to employees from winning a heterogeneous tournament is more than the prize they win from a homogeneous one. In the present model, however, learning of employee abilities is the driving force behind this result.

The remainder of the paper is organized as follows: The next section, i.e. Section 2, presents the model description. Section 3 demonstrates the optimality of designing heterogeneous promotion tournaments. Section 4 discusses the findings and Section 5 concludes. All proofs are relegated to the Appendix.

2 Description of the model and notation

We consider a model with two periods, $\tau = 1, 2$. There are n workers (or employees) and N firms (or employers) in the market. It is assumed that $n < N$, and that workers have the complete bargaining power (i.e. the labor market is competitive). All parties are risk-neutral. At the beginning of the first period, each firm $i \in \{1, 2, ..., N\}$ hires up to two workers whose output accrues to the firm. Output is nonobservable and depends on workers' abilities and their effort choices, $y_{i\tau} = \sum_{j_{i\tau}} (t_{j_{i\tau}} + e_{j_{i\tau}\tau})$, where $t_{j_{i\tau}}$ is ability, $e_{j_{i\tau}\tau} \geq 0$ is effort, and $j_{i\tau}$ are indexing workers employed by the firm i during period τ. At the outset, workers' abilities are not known exactly to all parties (i.e. there is symmetric uncertainty regarding abilities, as discussed for example by Holmström 1982). However, there are different types of workers and prior expectations regarding their abilities are different for different types.[4] In particular,

[4]For instance, in many situations it is reasonable to expect that workers with a college degree are more capable than workers who have only high school education.

the assumptions are that ability has three possible realizations, $0 \leq t_l < t_m < t_h$ and that workers are of two types: type 1 and type 2. Type 1 is either of middle or high ability (i.e. $t_1 \in \{t_m, t_h\}$) and type 2 of low or middle ability (i.e. $t_2 \in \{t_l, t_m\}$). Let $\delta = t_h - t_m = t_m - t_l$ and $P\{t_1 = t_m\} = P\{t_2 = t_l\} = 0.5$.[5] Worker types are common knowledge. There are $\frac{n}{2}$ workers of type 1 and $\frac{n}{2}$ workers of type 2.

If firm i hires two workers during period 1, their relative first-period performance can be measured by that firm, and also by all other potential employers (i.e. the labor market) at the end of the first period. The two workers are denoted by A and B and their relative performance by x_{A-B}. The probability that x_{A-B} is positive is denoted by p_A, which means that worker A has performed better than worker B. It is assumed that $p_A = p_A(t_A, t_B, e_{A1}, e_{B1})$ with $\frac{\partial p_A}{\partial t_A}, \frac{\partial p_A}{\partial e_{A1}} > 0$ and $\frac{\partial p_A}{\partial t_B}, \frac{\partial p_A}{\partial e_{B1}} < 0$, i.e. the signal realization depends on workers' efforts and their abilities. The labor market thus uses the performance signal to deduce workers' abilities. If firm i hires only one worker, the worker's performance cannot be measured.

At the beginning of the second period, all firms make second-period wage offers to the workers, after taking into account the first-period performance signals. Explicit incentive schemes, conditioning pay on performance, are not feasible. Furthermore, long-term contracts that bind workers to the firm for both periods are not feasible as well.

Effort is costly to the workers, and the corresponding effort costs are given by $c(e_{j_{i\tau}1}, e_{j_{i\tau}2})$. Costs are increasing, strictly convex and satisfy $\frac{\partial c(0,0)}{\partial e_{j_{i\tau}\tau}} = 0$ for $\tau = 1, 2$. Finally, there is no discounting.

3 Model solution

3.1 Preliminaries

The model is solved by backward induction. Because effort costs are increasing in the level of effort, and the firms cannot use pay-for-performance schemes, all workers who are hired during the second period choose second-period effort of zero, $e_{j_{i2}2} = 0$.[6] Hence, the output

[5] These assumptions ensure that there is always an equilibrium with symmetric effort choices.

[6] The fact that effort is zero is a normalization and should not be taken literally. This should be interpreted as the effort that the workers would choose if there were no (explicit or implicit) incentive pay. Typically, workers exert some "regular" effort level even in the absence of incentive pay, because they experience some utility from working up to a certain point. This regular effort level is normalized to zero in the model. A

by a worker for the firm is given by $t_{j_{i2}}$. Because the labor market is competitive, firms are constrained to earn zero expected profit, so that the worker's second-period wage is given by $E[t_{j_{i2}}]$, where $E[\cdot]$ denotes the expectation operator. Importantly, the higher the firms assess a worker's ability, the more they would be willing to pay to hire the worker for the second period.

Reverting to the first period, it follows that if a firm hires only one worker, the labor market does not learn anything about the worker's performance and, thus, his ability. This means that the worker's first-period effort has no effect on his second-period wage, and, as a result, the worker again finds it optimal to choose zero effort.

3.2 First-period tournaments

However, the argument would be different if the firm hires two workers in the first period. In this situation, the labor market observes which of the two workers performs better and uses this information to update the assessment of the two workers' abilities. Obviously, the workers have an incentive to affect the relative performance signal and their second-period compensation. Denoting the two workers again by A and B, worker A has an incentive to put forth effort in the first period, if and only if $E[t_A | x_{A-B} > 0] > E[t_A | x_{A-B} < 0]$. More precisely, his first-period effort is chosen so as to maximize

$$\begin{aligned} U_A &= E[t_A] - c(e_A, 0) \\ &= E[t_A | x_{A-B} < 0] + (E[t_A | x_{A-B} > 0] - E[t_A | x_{A-B} < 0]) p_A - c(e_A, 0). \end{aligned}$$

Obviously, $E[t_A | x_{A-B} > 0]$ and $E[t_A | x_{A-B} < 0]$ depend on whether the two workers are of the same type or of different types.

We assume that realization of the relative performance signal is strictly positive, $x_{A-B} > 0$, if and only if $t_A - t_B + e_{A1} - e_{B1} + \varepsilon > 0 \iff t_A - t_B + \varepsilon > e_{B1} - e_{A1}$, with ε being a random variable that is symmetrically distributed around zero, and which captures luck or measurement error. Denote by $F_{A,B}$ the cdf of the random variable $t_A - t_B + \varepsilon$ and by $e_{B-A} := e_B - e_A$ the effort difference between B and A. It follows that $F_{A,B}(e_{B1-A1})$ describes the probability of observing $x_{A-B} < 0$ (i.e. worker B performing better than worker A), given worker A's ability is t_A, worker B's ability is t_B and given the first-period efforts of e_{A1} and

similar argument is advanced by Holmström and Milgrom (1991) and Grund and Sliwka (2010).

e_{B1}. Let the respective pdf be given by $f_{A,B}$. Notice that $F_{h,m}(e_{B1-A1}) = F_{m,l}(e_{B1-A1})$ and $F_{h,h}(e_{B1-A1}) = F_{m,m}(e_{B1-A1}) = F_{l,l}(e_{B1-A1})$.

As explained before, the information transmitted by the performance signal depends on whether the two considered workers are of the same type or of different types. Therefore, the incentive to put forth effort depends on the workers' types as well. To start with, a heterogeneous tournament is considered, in which worker A is of type 1 and worker B of type 2. Worker A's (unconditional) probability of winning the tournament can be restated as

$$p_A^{het}(e_{B1-A1}) = 1 - 0.25\left[F_{m,l}(e_{B1-A1}) + F_{m,m}(e_{B1-A1}) + F_{h,l}(e_{B1-A1}) + F_{h,m}(e_{B1-A1})\right].$$

If the labor market believes that the two workers choose efforts $\tilde{e}_{A1}$ and $\tilde{e}_{B1}$, then, using Bayes' rule, the market perception about worker A's ability can be calculated in case he wins ($x_{A-B} > 0$) and also in case he loses ($x_{A-B} < 0$). These calculations are given by

$$\begin{aligned}
E\left[t_A | x_{A-B} > 0\right] &= t_m + \delta \frac{\Pr\{x_{A-B} > 0 | t_A = t_h\} \cdot \Pr\{t_A = t_h\}}{\Pr\{x_{A-B} > 0\}} \\
&= t_m + 0.5\delta \frac{1 - 0.5F_{h,l}(\tilde{e}_{B1-A1}) - 0.5F_{h,m}(\tilde{e}_{B1-A1})}{p_A^{het}(\tilde{e}_{B1-A1})}, \\
E\left[t_A | x_{A-B} < 0\right] &= t_m + 0.5\delta \frac{0.5F_{h,l}(\tilde{e}_{B1-A1}) + 0.5F_{h,m}(\tilde{e}_{B1-A1})}{1 - p_A^{het}(\tilde{e}_{B1-A1})}.
\end{aligned}$$

Similar calculations can be made worker B:

$$\begin{aligned}
E\left[t_B | x_{A-B} < 0\right] &= t_l + 0.5\delta \frac{0.5F_{m,m}(\tilde{e}_{B1-A1}) + 0.5F_{h,m}(\tilde{e}_{B1-A1})}{1 - p_A^{het}(\tilde{e}_{B1-A1})}, \\
E\left[t_B | x_{A-B} > 0\right] &= t_l + 0.5\delta \frac{1 - 0.5F_{m,m}(\tilde{e}_{B1-A1}) - 0.5F_{h,m}(\tilde{e}_{B1-A1})}{p_A^{het}(\tilde{e}_{B1-A1})}.
\end{aligned}$$

Define $\Delta_A^{het} := E\left[t_A | x_{A-B} > 0\right] - E\left[t_A | x_{A-B} < 0\right]$ and $\Delta_B^{het} := E\left[t_B | x_{A-B} < 0\right] - E\left[t_B | x_{A-B} > 0\right]$. As indicated before, the two workers choose first-period efforts in order to maximize

$$\begin{aligned}
U_A^{het} &= E\left[t_A | x_{A-B} < 0\right] + \Delta_A^{het} p_A^{het}(e_{B1-A1}) - c(e_{A1}, 0), \\
U_B^{het} &= E\left[t_B | x_{A-B} > 0\right] + \Delta_B^{het}\left(1 - p_A^{het}(e_{B1-A1})\right) - c(e_{B1}, 0).
\end{aligned}$$

As first-order conditions, the following are obtained:[7]

$$\Delta_A^{het} \frac{\partial p_A^{het}\left(e_{B1}^{het}-e_{A1}^{het}\right)}{\partial e_{A1}} = \frac{\partial c\left(e_{A1}^{het},0\right)}{\partial e_{A1}},$$
$$\Delta_B^{het} \frac{\partial\left(1-p_A^{het}\left(e_{B1}^{het}-e_{A1}^{het}\right)\right)}{\partial e_{B1}} = \frac{\partial c\left(e_{B1}^{het},0\right)}{\partial e_{B1}}.$$

Because the market correctly anticipates the workers' behavior, $\tilde{e}_{A1} = e_{A1}^{het}$ and $\tilde{e}_{B1} = e_{B1}^{het}$. Taking into account all these conditions, the following lemma is obtained.

Lemma 1 *In the heterogeneous tournament, a unique equilibrium exists that is symmetric with both workers choosing the same effort,* e^{het}*, which is implicitly defined by*

$$\frac{\partial c\left(e^{het},0\right)}{\partial e^{het}} = \delta \cdot \frac{\left(2f_{h,m}(0)+f_{m,m}(0)+f_{h,l}(0)\right)\cdot\left(1-2F_{h,l}(0)\right)}{16-\left(3-4F_{h,m}(0)-2F_{h,l}(0)\right)^2}.$$

The analysis of the homogeneous tournament involving two workers either of type 1 or of type 2, proceeds exactly the same way as the preceding analysis. The following lemma is the counterpart of Lemma 1.

Lemma 2 *In the homogeneous tournament, a unique equilibrium exists that is symmetric with both workers choosing effort,* e^{hom}*, which is implicitly defined by*

$$\frac{\partial c\left(e^{hom},0\right)}{\partial e^{hom}} = \delta \cdot \frac{\left(2f_{m,m}(0)+f_{l,m}(0)+f_{m,l}(0)\right)\cdot\left(1-2F_{m,l}(0)\right)}{16-\left(1-2F_{m,l}(0)\right)^2}.$$

By the convexity of c, it follows that

$$e^{het} > e^{\text{hom}}$$

$$\Leftrightarrow \frac{2f_{h,m}(0)+f_{m,m}(0)+f_{h,l}(0)}{2f_{m,m}(0)+f_{l,m}(0)+f_{m,l}(0)} > \frac{\frac{1-2F_{m,l}(0)}{16-\left(1-2F_{m,l}(0)\right)^2}}{\frac{1-2F_{h,l}(0)}{16-\left(1-2F_{h,l}(0)+2\left(1-2F_{h,m}(0)\right)\right)^2}}. \qquad (1)$$

This condition nicely displays the two different effects that worker heterogeneity has on the incentive to put forth effort. On the one hand, heterogeneity affects the marginal effect of effort on the probability of winning. As demonstrated in the existing tournament literature, this effect is non-positive, that is heterogeneity reduces the incentive to put forth effort by

[7] A typical feature of tournament models is that the cost function must be sufficiently convex for the objective function to be strictly concave and for meeting the second-order conditions. In what follows, it is assumed that this is the case so that optimal efforts are indeed characterized by the first-order conditions to the maximization problem.

reducing the marginal effect of effort on the probability of winning. Formally, the effect is captured by the term on the left-hand-side of the inequality and the observation that $2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)$ is typically not greater than $2f_{m,m}(0) + f_{l,m}(0) + f_{m,l}(0)$.[8] On the other hand, worker heterogeneity has an effect on how much information is transmitted to the labor market through relative performance signal. For instance, if a worker, who is thought to have low ability, performs better than a worker of seemingly higher ability, the worker's ability assessment is upgraded to a degree stronger than the one had he performed better than another worker of rather low ability. In turn, when the labor market puts greater emphasis on the performance measure, the workers have a higher incentive to put forth effort to affect the performance measure and, thus, their ability assessment and future compensation. This is captured by the term on the right-hand-side of the inequality.

To be able to evaluate the size of the single effects, more structure is put on the model by assuming some properties of the distribution. This results in the following proposition, which states the main result of this paper: worker heterogeneity leads to higher incentives, because it changes the way the labor market uses the performance signal to update workers' abilities.

Proposition 1 *Let ε be distributed according to density f that is symmetric around zero and has a global maximum at zero. Further, assume that $f(e+\delta)/f(e)$ as well as $f(e)\cdot(1-F(e))$ (weakly) decrease in e on $[-2\delta, -\delta]$. Then, workers' effort is higher in the heterogeneous tournament with workers of different types than in the homogeneous tournament with workers of the same type.*

Proposition 1 provides conditions that are sufficient for workers to put forth higher effort in a heterogeneous tournament than in a homogeneous one. It is easy to demonstrate that the conditions are met for many of the standard distributions. The first condition (regarding $f(e+\delta)/f(e)$) corresponds to the well-known monotone likelihood ratio property (Milgrom 1981) and is satisfied for most of the standard distributions, including the uniform, the triangular, and the normal distribution. This holds true for the second condition (regarding

[8] Typically, the variable ε is assumed to be distributed according to a pdf that is symmetric around zero and that has a global maximum at zero. Formally, this means that $2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)$ is not greater than $2f_{m,m}(0) + f_{l,m}(0) + f_{m,l}(0)$ (which can be restated as $2f_{h,m}(0) + f_{m,m}(0) + f_{m,m}(0)$ because $f_{m,l}(0) = f_{l,m}(0) = f_{h,m}(0)$).

$f(e)\cdot(1-F(e)))$ too. This condition is fulfilled, for example if ε is uniformly distributed on $[-u,u]$ (with $2\delta<u$) because in this case f is constant and F increases strictly on $[-2\delta,-\delta]$. Also, the triangular and the normal distributions (both with zero mean) satisfy this condition if the ratio between δ and the standard deviation is sufficiently small.

3.3 Matching of workers to firms

In this subsection, it is analyzed how workers are matched to firms at the beginning of the first period. Imposing the conditions of Proposition 1, we have demonstrated that workers choose a relatively higher effort if the other worker hired along with them is of a different type, rather than of the same type. Of course, inducing higher effort is beneficial only if effort is not inefficiently high already. Given that the second-period effort is zero, the first-period effort e^* that maximizes total surplus is characterized by the first-order condition $(\partial c/\partial e)(e^*,0)=1$. To ensure that equilibrium efforts are never inefficiently high (so that $e^{\text{hom}}<e^{het}\leq e^*$), the following additional assumption is made:

$$(A1)\quad \delta\cdot\frac{(2f_{h,m}(0)+f_{m,m}(0)+f_{h,l}(0))\cdot(1-2F_{h,l}(0))}{16-(3-4F_{h,m}(0)-2F_{h,l}(0))^2}\leq 1.$$

The following proposition then demonstrates that the firms decide to hire heterogeneous workers and, thus, to implement heterogeneous tournaments. This is because heterogeneous tournaments enable the firms to induce the highest efforts, i.e. efforts that are closest to the efficient level.

Proposition 2 *Suppose that the conditions from Proposition 1 hold and let assumption (A1) be fulfilled. Then in equilibrium, $\frac{n}{2}$ of the N firms each hire one worker of type 1 and one worker of type 2, while the remaining firms do not hire a worker at all. Workers of type 1 are paid a first-period wage of $w_1=t_m+0.5\delta+e^{het}$ and workers of type 2 a wage of $w_2=t_l+0.5\delta+e^{het}$.*

As workers of different types receive different first and second-period wages, the change in compensation (relative to that of the first period), following a win or loss in the tournament may be different for the workers as well. Let the difference between the second-period and first-period wages of a worker of type 1, after winning the tournament, be denoted by Δ_1, i.e. $\Delta_1=E[t_A|\,x_{A-B}>0]-w_1$, and let Δ_2 be defined analogously, i.e. $\Delta_2=E[t_B|\,x_{A-B}<0]-$

w_2. It is straightforward to demonstrate that

$$\begin{aligned}\Delta_1 &= \delta\frac{0.125\left[F_{m,m}(0)-F_{h,l}(0)\right]}{p_A^{het}(0)}-e^{het} \text{ and}\\ \Delta_2 &= \delta\frac{0.125\left[F_{m,m}(0)-F_{h,l}(0)\right]}{1-p_A^{het}(0)}-e^{het}.\end{aligned}$$

Since $p_A^{het}(0)>0.5>1-p_A^{het}(0)$ and $F_{m,m}(0)>F_{h,l}(0)$, the following proposition is immediate:

Proposition 3 *Suppose that the conditions from Proposition 1 hold and let assumption (A1) be fulfilled. Then in equilibrium we observe* $\Delta_1<\Delta_2$. *This means that the change in compensation after winning the tournament (relative to the first-period compensation) is higher for a worker of type* 2 *than for a worker of type* 1.

Proposition 1 demonstrates that the incentive to put forth effort may be higher in a heterogeneous tournament than in a homogeneous one. Proposition 3 sheds more light on this issue. The proposition shows that workers of type 2 (i.e. workers with relatively low ability) have a high incentive to win the tournament, because winning against a high-ability worker leads to a significant upgrade in the ability assessment and, accordingly, to a higher wage. On the contrary, the ability assessment of type 1-worker (i.e. a worker of rather high ability) does not change significantly if he competes successfully against a low-ability worker. Instead, the worker is motivated to put forth effort, because his ability assessment were strongly downgraded if the worker would lose against a worker of low ability. Thus, the worker strives hard to avoid not being successful.

4 Discussion

Recently, there has been a discussion in the tournament literature about the empirical relevance of "classic tournaments" in the spirit of Lazear and Rosen (1981) relative to that of "market-based tournaments" analyzed in the current paper. DeVaro (2011) and Waldman (forthcoming) discuss possible ways to empirically differentiate between these two types of tournaments. The model proposed here opens up another possibility to differentiate between classic and market-based tournaments. Whereas in classic tournaments workers decrease their effort as a response to higher worker heterogeneity, the current paper shows that, in market-based tournaments, the reaction of workers who take part in homogeneous tournaments may be opposite in that they choose lower effort than those who take part in heterogeneous tournaments. Thus, by analyzing the reaction of contestants to heterogeneity, the empirical relevance of both types of tournament can be assessed.

Some empirical evidence exists that is in line with the findings of the present model. First, in sports there is ample anecdotal evidence of young and relatively unknown athletes "becoming stars overnight" after succeeding against well-known rivals. Included among them are the famous tennis players Boris Becker and Roger Federer who won the Wimbledon men's title in 1985 and 2003, respectively, and basketball player Jeremy Lin who scored 38 points and proved himself instrumental in the victory of the New York Knicks against the Los Angeles Lakers in 2012. These findings indicate that performing well against strong rivals can indeed boost a player's career, thus substantiating the finding of Proposition 1 that the information contained in relative performance signals is more significant in the case of contestants who are heterogeneous. Further support to the findings of this paper is provided by the empirical studies of DeVaro and Waldman (2012) and Bognanno and Melero (2012). DeVaro and Waldman (2012), based on the data of a medium-sized US firm in the financial-services industry, find that employees with a Masters degree or a Ph.D. receive, on promotion, a smaller wage increase as compared to that of employees with a Bachelors degree. This observation is consistent with Proposition 3 of this paper, in which it is demonstrated that, upon winning the tournament, the ability assessment of low-ability employees is upgraded to a stronger degree relative to that of high-ability employees. Bognanno and Melero (2012), using data from the British Household Panel Survey, obtain similar results: less educated employees receive larger wage increases on promotion.

Some empirical studies analyze the effects of contestant heterogeneity in tournaments. In contrast to the current model, these studies seem to suggest that heterogeneity of contestants leads to a lower effort, because of which the tournament organizer tries to avoid too heterogeneous contestants. Brown (2011) demonstrates that PGA golfers need on average 0.2 additional strokes to complete the first round of the golf course when Tiger Woods participates in the tournament relative to when he is absent. This can be understood as evidence of lower effort provision as a response to higher player heterogeneity. Knoeber and Thurman (1994) study broiler production, where producers contract with growers to raise their broiler chickens and reward them depending on their performance relative to other growers. Knoeber and Thurman find evidence that is in line with the ideas of producers handicapping growers of high ability and of producers sorting growers into homogeneous tournaments. At first sight, these results seem to be inconsistent with the model presented here. However, it is to be noted that the present model applies to rather young contestants, about whom initially little is known so that the relative performance signal is used to deduce contestants' abilities. The preceding studies consider not only young contestants, but also all contestants regardless of their age. An empirical test of the model requires that the field of contestants be segregated by age. For young contestants, learning of ability is important so that contestant heterogeneity increases the incentive to put forth effort, as shown in the present model. In contrast, the characteristics of older contestants may already be well-known. Here, heterogeneity is expected to have a negative impact on effort, as demonstrated in "classic tournament models".

5 Conclusion

In this paper, we investigate market-based tournaments, in which firms use relative performance signals to estimate workers' abilities. Workers have an incentive to put forth effort in order to win the tournament, because winning has a positive impact on the ability assessment. It is demonstrated that firms learn more from relative performance signal when workers are heterogeneous than when they are homogeneous. Therefore, in a heterogeneous tournament, workers exert higher effort, because they have a stronger incentive to affect the tournament outcome. Hiring heterogeneous workers may then be optimal for firms.

More generally, the latter finding implies that policies, which are aimed at "leveling the playing field", are not always as beneficial as they may appear. If workers succeed in spite of many obstacles, the labor market learns a lot about their characteristics, based on which it can reward the workers generously. This may induce workers to put forth much greater effort than when the playing field is a leveled one.

Appendix

Proof of Lemma 1. On one hand, we have $\Delta_A^{het}(e) = \Delta_B^{het}(e)$ for all e because

$$\begin{aligned}
\Delta_A^{het}(e) &= 0.5\delta \left(\frac{1 - 0.5F_{h,l}(e) - 0.5F_{h,m}(e)}{p_A^{het}(e)} - \frac{0.5F_{h,l}(e) + 0.5F_{h,m}(e)}{1 - p_A^{het}(e)} \right) \\
&= 0.5\delta \left(\frac{1 - 0.5F_{h,l}(e) - 0.5F_{h,m}(0) - p_A^{het}(e)}{p_A^{het}(e)\left(1 - p_A^{het}(e)\right)} \right) \\
&= 0.5\delta \left(\frac{1 - 0.5F_{h,l}(e) - 0.5F_{h,m}(e) - 1 + 0.25\left(2F_{h,m}(e) + F_{m,m}(e) + F_{h,l}(e)\right)}{p_A^{het}(e)\left(1 - p_A^{het}(e)\right)} \right) \\
&= 0.125\delta \left(\frac{F_{m,m}(e) - F_{h,l}(e)}{p_A^{het}(e)\left(1 - p_A^{het}(e)\right)} \right)
\end{aligned}$$

and

$$\begin{aligned}
\Delta_B^{het}(e) &= 0.5\delta \left(\frac{0.5F_{m,m}(e) + 0.5F_{h,m}(e)}{1 - p_A^{het}(e)} - \frac{1 - 0.5F_{m,m}(e) - 0.5F_{h,m}(e)}{p_A^{het}(e)} \right) \\
&= 0.5\delta \left(\frac{0.5F_{h,m}(e) + p_A^{het}(e) - 1 + 0.5F_{m,m}(e)}{p_A^{het}(e)\left(1 - p_A^{het}(e)\right)} \right) \\
&= 0.5\delta \left(\frac{0.5F_{h,m}(e) + 1 - 0.25\left(2F_{h,m}(e) + F_{m,m}(e) + F_{h,l}(e)\right) - 1 + 0.5F_{m,m}(e)}{p_A^{het}(e)\left(1 - p_A^{het}(e)\right)} \right) \\
&= 0.125\delta \left(\frac{F_{m,m}(e) - F_{h,l}(e)}{p_A^{het}(e)\left(1 - p_A^{het}(e)\right)} \right).
\end{aligned}$$

On the other hand and under consideration of

$$\frac{\partial p_A^{het}}{\partial e_{A1}} = 0.25\left(f_{m,l}(e_{B1-A1}) + f_{m,m}(e_{B1-A1}) + f_{h,l}(e_{B1-A1}) + f_{h,m}(e_{B1-A1})\right) = \frac{\partial\left(1 - p_A^{het}\right)}{\partial e_{B1}}$$

the first-order conditions imply $sgn\left(e_{A1} - e_{B1}\right) = sgn\left(\Delta_A^{het}(e_{B1} - e_{A1}) - \Delta_B^{het}(e_{B1} - e_{A1})\right)$. Because the latter term is zero, the first-order conditions lead to $e_{A1} = e_{B1}$. This means that the equilibrium is unique and symmetric. The rest of the lemma follows because

$$\begin{aligned}
\frac{\partial c\left(e^{het}, 0\right)}{\partial e^{het}} &= \Delta_A^{het} \left. \frac{\partial p_A^{het}\left(e_{B1}^{het} - e_{A1}^{het}\right)}{\partial e_{A1}} \right|_{e_{A1}^{het} = e_{B1}^{het} = e^{het}} \\
&= 0.25\left(2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)\right) \cdot \frac{\delta}{16} \cdot \frac{1 - 2F_{h,l}(0)}{p_A^{het}(0)\left(1 - p_A^{het}(0)\right)}. \\
&= \delta \cdot \frac{\left(2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)\right) \cdot \left(1 - 2F_{h,l}(0)\right)}{\left(1 + 4F_{h,m}(0) + 2F_{h,l}(0)\right)\left(7 - 4F_{h,m}(0) - 2F_{h,l}(0)\right)} \\
&= \delta \cdot \frac{\left(2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)\right) \cdot \left(1 - 2F_{h,l}(0)\right)}{16 - \left(3 - 4F_{h,m}(0) - 2F_{h,l}(0)\right)^2}.
\end{aligned}$$

■

Proof of Lemma 2. Suppose that both workers are of type 2. Then the analysis is the same as the analysis of the heterogeneous tournament except that $f_{h,m}$ must be replaced by $f_{m,m}$ (since A is no longer of type 1 but type 2), $f_{m,m}$ by $f_{l,m}$, $f_{h,l}$ by $f_{m,l}$, $F_{h,m}$ by $F_{m,m}$, and $F_{h,l}$ by $F_{m,l}$. Considering these changes, we immediately obtain the expression for optimal effort. Because $f_{m,m} = f_{h,h}$, $f_{l,m} = f_{m,h}$, $f_{m,l} = f_{h,m}$, $F_{h,m} = F_{m,l}$, and $F_{h,h} = F_{m,m}$, the effort is the same if both workers are of type 1. ■

Proof of Proposition 1. Due to

$$\frac{16 - \left(1 - 2F_{h,l}(0) + 2\left(1 - 2F_{h,m}(0)\right)\right)^2}{16 - \left(1 - 2F_{m,l}(0)\right)^2} < \frac{16 - \left(1 - 2F_{h,l}(0)\right)^2}{16 - \left(1 - 2F_{m,l}(0)\right)^2} < 1$$

and

$$\frac{2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)}{2f_{m,m}(0) + f_{l,m}(0) + f_{m,l}(0)} = \frac{2f_{h,m}(0) + f_{m,m}(0) + f_{h,l}(0)}{2f_{h,m}(0) + f_{m,m}(0) + f_{m,m}(0)} > \frac{f_{h,l}(0)}{f_{m,m}(0)}$$

equation (1) is fulfilled if

$$\frac{f_{h,l}(0)}{f_{m,m}(0)} \geq \frac{1 - 2F_{m,l}(0)}{1 - 2F_{h,l}(0)}. \tag{2}$$

The assumptions of the proposition imply

$$f_{m,m}(-2\delta)(1 - F_{m,m}(-2\delta)) \geq f_{m,m}(-\delta)(1 - F_{m,m}(-\delta)) \quad \Leftrightarrow \quad \frac{f_{h,l}(0)}{f_{m,l}(0)} \geq \frac{1 - F_{m,l}(0)}{1 - F_{h,l}(0)}$$

and

$$\frac{f_{m,m}(0)}{f_{m,m}(-\delta)} \leq \frac{f_{m,m}(-\delta)}{f_{m,m}(-2\delta)} \quad \Leftrightarrow \quad \frac{f_{m,l}(0)}{f_{m,m}(0)} \geq \frac{f_{h,l}(0)}{f_{m,l}(0)}$$

which immediately leads to (2) because

$$\frac{f_{h,l}(0)}{f_{m,m}(0)} = \frac{f_{h,l}(0)}{f_{m,l}(0)} \cdot \frac{f_{m,l}(0)}{f_{m,m}(0)} \geq \frac{(1 - F_{m,l}(0))^2}{(1 - F_{h,l}(0))^2} = \frac{1 - 2F_{m,l}(0) + F_{m,l}^2(0)}{1 - 2F_{h,l}(0) + F_{h,l}^2(0)} \geq \frac{1 - 2F_{m,l}(0)}{1 - 2F_{h,l}(0)}.$$

The latter inequality results because $F_{m,l}^2(0) \geq F_{h,l}^2(0)$ and $1 - 2F_{m,l}(0) < 1 - 2F_{h,l}(0)$. ■

Proof of Proposition 2. The proof is divided into two steps.

Step (i): In step (i) we demonstrate that in equilibrium there is no firm that hires a single worker in the first period. The proof is by way of contradiction. Suppose there is a firm that hires a single worker. Then, because the number of workers is even, there must either be another firm that has hired only one worker or at least one worker has not been hired at all. In either situation, the worker of the considered firm and the other worker choose zero first-period effort. Suppose first that all workers have been hired, and denote the first-period wage that the alternative (single) worker obtains at his firm by $\hat{w}$. Because the labor market is competitive and the worker chooses zero effort, this wage equals the worker's expected ability, $\hat{w} = E[t]$. If the initially considered firm would hire the worker away from the alternative firm, the firm would now employ two workers and the workers would thus choose positive effort, $e \in \{e^{\text{hom}}, e^{het}\}$ depending on whether or not the tournament is homogeneous or heterogeneous. In this case, the firm would have to increase the own worker's wage by $c(e, 0)$ to compensate him for the additional effort costs he has to bear. Moreover, to hire the second worker away from the alternative firm (and to ensure that this firm is not interested in making a counteroffer), the firm had to pay the worker a wage of slightly more than $\hat{w}+c(e, 0)$. Then, the firm's additional profit would be close to $2e+E[t]-c(e, 0)-\hat{w}-c(e, 0)$, or $2(e-c(e, 0))$, which is strictly positive since e is below the efficient level of effort. This means that the firm has an incentive to deviate from its hiring strategy and to hire a second worker in addition to the worker currently employed. A very similar argument applies if one worker is not hired at all.

Step (ii): Step (i) implies that there is no firm that hires only one worker. Furthermore, because workers represent the short side of the market, there is no worker that is not hired at all. Together, these two results indicate that $\frac{n}{2}$ firms hire two workers each, whereas the remaining firms do not hire a worker at all. It remains to be shown that each firm decides to hire heterogeneous workers of different types. Again the proof proceeds by way of contradiction. If there is one firm that hires two workers of the same type, say type 1, there must be another firm that hires two workers of type 2. Then, each of the two firms would gain from deviating from the hiring strategy by not employing one of its current workers and instead hiring one of the workers away from the alternative firm such that a heterogeneous tournament is created. Since efforts in the heterogeneous tournament are closer to the efficient level, the firm could compensate the workers for the additional effort cost, ensure that the other firm is not willing to make a counteroffer for the targeted worker and still profit from the change in hiring policy. Therefore, the initial situation does not represent an equilibrium.

Step (iii): In the previous two steps we have demonstrated that $\frac{n}{2}$ of the N firms each hire one worker of type 1 and one worker of type 2. Since the labor market is competitive, firms earn zero profit so that each worker receives a first period wage equal to the expected output level that the worker produces for the firm. Hence, a worker of type 1 is paid a first-period wage of $w_1 = t_m + 0.5\delta + e^{het}$ and a worker of type 2 a wage of $w_2 = t_l + 0.5\delta + e^{het}$. ■

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